Aristo Nr. 10065 System Gruter
Operation and Examples
Richard Smith Hughes
PRECISION SLIDE

Multiplication, Division; Three Digit + Estimate Accuracy (Minimum)
Log, Antilog; Four Figure Accuracy
**Multiplication and Division**

**Multiplication**

\[(a)(b) = (8.2736)(9.5927) = 79.36616\]

1) Determine the decimal point;

\[(8)(10) = 80\]

2) Find the solution scale. Using the C/D scales solve for \( (a)(b) = 79.3 \); the solution will be found on scale 8, 630 to 795.

**Slide Rule Operation**

a) Initial Position; A/B, C/D indexes aligned.

b) Move the cursor to \( a = 82736 \) on scale 9.

c) Bring the C/B index (left) under the cursor.

d) Move the cursor to \( b = 95927 \) on scale 9.

e) Initial Position; A/B, C/D indexes aligned.

f) Read \( (a)(b) \) on scale 8 (see 2 above) = 79.38.

g) Error = 0.02%.

**Division**

\[a/b = 9.8285/8.8359 = 1.112337\]

1) Determine the decimal point;

\[10/10 = 1\]

2) Find the solution scale. Using the C/D scales solve for \( a/b = 1.11 \); the solution will be found on scale 0, 100 to 126.

**Slide Rule Operation**

a) Initial Position; A/B, C/D indexes aligned.

b) Move the cursor to \( a = 98285 \) (scale 9).

c) Bring \( b = 88359 \) (scale 9) under the cursor.

d) Move the cursor to the C/B (left) index.

e) Initial Position; A/B, C/D indexes aligned.

f) Read \( (a)/(b) \) on scale 0 (see 2 above) = 1.1121.

g) Error = -0.02%.

**Logs and Anti Logs**

\[\log_{10} y = + N\]

A) \( y>1 \) (\( N>1 \), positive); \( y = 982,460 \)

Write \( y \) as \( Y.yyyE^x \) (Note; \( E^x = 10^x \) = 9.8246E^5

\[N = \log_{10} y = \log_{10}(Y.yyyE^x) = \log_{10}E^x + \log_{10}(Y.yyy) = x + \log_{10}(Y.yyy) = \]

Characteristic C (\( C = x \)) + Mantissa M (\( M = 0.mmm = \log_{10}(Y.yyy) \))

\[y = Y.yyyE^x = 9.8246E^5\]

\[N = \log_{10}(9.8246E^5) = \log_{10}(Y.yyy E^x) = \log_{10}E^5 + \log_{10}(9.8246)\]

Characteristic C (\( C =5 \)) + Mantissa M (\( M = 0.mmm = \log_{10}(9.8246) = 5 + 0.99231\)
Log\(_{10}(9.8246 \times 10^{-5}) = 5 + 0.99231 = 5.99231\)

Slide Rule Operation

a) Initial Position; A/B, C/D indexes aligned.

b) Move the cursor to Y.yyyy = 9.8246 on scale 9 (this scale number, 9, is the most significant mantissa number; M = 0.mmm = 0.9mmm).

c) On the L scale read the remaining mantissa numbers, 0.09238.

d) The mantissa M = 0.mmm = 0.99238.

e) Log\(_{10}(9.8246 \times 10^5) = C + M = 5 + 0.99238 = 5.99238\).

f) Error = 0.001%.

\(y = 10^N\)

B) N>0, positive (y>1); N = 5.99231

Write N as C (characteristic) + M (mantissa = 0.mmm)

\[ y = 10^N = [10^C][10^0.mmm] = 10^0.mmmE^C \]

\[ N = 5.99231 = \text{Characteristic (C = 5) + Mantissa M (0.mmm = 0.99231)} \]

\[ y = 10^{5.99231} = [10^{0.99231}][10^5] = 9.8245 \times 10^5 \]

Slide Rule Operation

a) The most significant mantissa number is 9 and \(10^{0.mmm} = 10^{0.99231}\) will be found on this scale, 9.

b) Place the cursor over the remaining mantissa numbers, 0.09231, on the L scale.

c) Initial Position; A/B, C/D indexes aligned.

d) Under the cursor on scale 9, remember this is the most significant mantissa number, read \(10^{0.mmm} = 10^{0.99231} = 9.824\).

e) \(10^C = E^5\).

f) \(y = 10^{5.99231} = 10^{0.99231+48}E^5 = 9.824E^5\).

g) Error = -0.005%.

\(\log_{10} y = -N\)

C) 0<y<1 (N<0, negative); y = 9.8246E^-5

Write y = Y.yyyyE^-x = 9.8246E^-5

\[ N = \log_{10}(9.8246E^{-5}) = \log_{10}(Y.yyyyE^{-x}) = \log_{10}E^{-5} + \log_{10}(9.8246) = \]

Characteristic (C = -5) + Mantissa (M = 0.mmm = \log_{10}(9.8246) = -5 + 0.99231)

\[ \log_{10}(9.8246E^{-5}) = -5 - 0.99232 = -4.00769 \]

Slide Rule Operation

a) Initial Position; A/B, C/D indexes aligned.

b) Move the cursor to Y.yyyy = 9.8246 on scale 9 (this scale number, 9, is the most significant mantissa number; M = 0.mmm = 0.9mmm).

c) On the L scale read the remaining mantissa numbers, 0.0924.

d) The mantissa M = 0.mmm = 0.9924.

e) The characteristic C = Log\(_{10}E^5 = -5\).
f) \[ N = \log_{10}(9.8246 \times 10^{-5}) = C + M = -5 + 0.9924 = -4.0076. \]
g) Error = 0.002%.

\[ y = 10^{-N} \]

D) \( N < 0 \), negative (0 < \( y < 1 \)); \( N = -4.00769 \)

Since \( N \) is negative, \(-N \neq \) Characteristic (C) + Mantissa M (0.mmm). We must find the characteristic, C, and mantissa, M = 0.mmm

Write \(-N = -N.nnnn\)

Characteristic \( C = -(1 + \vert N \vert) \)

Mantissa \( M = 0.mmm = (1 - 0.nn) \)

\[ N = -4.00769 = N.nnn = C [-(1 + \vert N \vert)] + M [(1 - 0.nn)] = \]

\[ C [-1 + \vert -4 \vert] = -5 + M [(1 - 0.00769) = 0.99231] = -5 + 0.992315 = -4.0769 \]

Now; \[ y = 10^{-4.00769} = 10^C 10^M = [10^{-5}][10^{0.992315}] = 9.8245E^{-5} \]

Slide Rule Operation

a) The most significant mantissa number is 9; \( y = 10^{-N} \) will be found on this scale, 9.
b) Place the cursor over the remaining mantissa numbers, 0.092315, on the L scale.
c) Initial Position; A/B, C/D indexes aligned.
d) On scale 9 read \( 10^{0.mmm} = 10^{0.992315} = 9.824 \) on scale 9.
e) \( 10^C = 10^{-5} = E^{-5} \).
f) \( y = 10^{-N} = 10^C 10^M = [10^{-5}][10^{0.992315}] = 9.824E^{-5} \).
g) Error = -0.006%.

Precision Slide Accuracy

<table>
<thead>
<tr>
<th>Multiplication</th>
<th>(8.2736)(9.5927) = 79.36616; Precision Slide = 79.38</th>
<th>Accuracy = 0.02%</th>
</tr>
</thead>
<tbody>
<tr>
<td>Division</td>
<td>9.8285/8.8359 = 1.112337; Precision Slide = 1.1121</td>
<td>Accuracy = -0.02%</td>
</tr>
<tr>
<td>Logs; ( N = \log_{10}y )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( y \geq 1 )</td>
<td>Actual</td>
<td>Precision Slide</td>
</tr>
<tr>
<td>9.8246E^{5}</td>
<td>5.99231</td>
<td>5.99238</td>
</tr>
<tr>
<td>( y &lt; 1 )</td>
<td>Actual</td>
<td>Precision Slide</td>
</tr>
<tr>
<td>9.8246E^{-5}</td>
<td>-4.00769</td>
<td>-4.0076</td>
</tr>
</tbody>
</table>

Antilogs; \( y = 10^{N} \)

| \( N \) | Actual | Precision Slide | Accuracy |
| 5.99231 | 9.8245E^{5} | 9.824E^{5} | -0.005% |
| -4.00769 | 9.8245E^{5} | 9.824E^{5} | -0.006% |
Slide I
Log Log

Scales
Front Side
K/A//B/LL01/LL02/LL03/C//D/L

Back Side
Slide I
Log Log

Tan Scale
The tan scale only goes to 45°. For angles greater than 45° simply remove the C scale and invert; a manual CI scale.

Log Log Scales
The Initial position must be used solving $\log_a y = N$ and $y = 10^N$
1) $\log_a y = N$; $\log_{10} 37.5 = 1.574031$
   a) Initial position, C/D and A/B indexes aligned
   b) Move the cursor to $y = 37.5$ on the LL3 scale
   c) Move the slide such the base $a = 10$ is under the cursor
   d) Read $\log_a y = N = \log_{10} 37.5 = 1.575$ under the C index on the D scale

2) $y = a^N$; $y = 4.2^{3.8} = 233.533$
   a) Move the C index to $N = 3.8$ on the D scale
   b) Move the cursor to $a = 4.2$ on the LL3 scale
   c) Initial position, C/D and A/B indexes aligned
   d) Read $y = 10^N = 4.2^{3.8} = 233.5$ on the LL3 scale

To solve $\log_e y = N$ and $e^N = y$, leave the slide in the Initial position and solve in the normal manner.
Note the E-R scale; for correcting the vertical height, $V$, due to the earth's curvature and refraction.

Red.hoz = Reduction to Horizontal
### Table 1
Stadia/Topographic Equations

#### Stadia Method

- **K** \( \equiv f/i = 100 \)
- **S** = Stadia rod reading \( = B - A \) (in feet or meters); **C** = 0
- **Di** = Line of sight distance \( = 100S(\cos \alpha) \)

**Reduction to Horizontal, Red.hoz**

- \( c = DI \) and \( \alpha \) are known
- \( b = H = c - x = DI - \Delta DI = DI - DI(\sin \alpha)(\tan \alpha/2) \)
- \( a = V = DI \sin \alpha \)

#### Horizontal Distance, \( H \)

<table>
<thead>
<tr>
<th>( 40^\circ &gt; \alpha &gt; 35' )</th>
<th>( 40^\circ &gt; \alpha &gt; 10^\circ )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( H = 100S - \Delta H )</td>
<td>( H = 100S(\cos^2 \alpha) )</td>
</tr>
<tr>
<td>( \Delta H = 100S(\sin^2 \alpha) )</td>
<td></td>
</tr>
</tbody>
</table>

#### Scales Used

<table>
<thead>
<tr>
<th>S/ST/A/B</th>
<th>Scale Index; B scale Index</th>
<th>( \cos^2 \alpha )/A</th>
<th>Scale Index; ( 0^\circ ) on ( \cos^2 ) scale</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>( \Delta H )</td>
<td>100S ( &gt; H &gt; 0.59(100S) )</td>
</tr>
<tr>
<td>0° 35’</td>
<td>0.0001(100S)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1° 48.7’</td>
<td>0.001(100S)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>5° 44.4’</td>
<td>0.01(100S)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>18° 25.4’</td>
<td>0.1(100S)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>40°</td>
<td>0.59(100S)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

#### Horizontal Distance, \( H \)

**Elevation (Vertical Height), \( V \)**

- \( \text{Di and } \alpha \text{ known} \)
- \( H = \text{Di} - \Delta \text{Di} \)
- \( \Delta \text{Di} = \text{Di}(\sin \alpha)(\tan \alpha/2) \)

#### Scales Used

<table>
<thead>
<tr>
<th>S/ST/C/D/Red.hoz, 0/Red hoz. 0°</th>
<th>( V = \text{Di}(\sin \alpha) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Index; C scale index</td>
<td>Index; ( 0^\circ ) on ( \cos^2 ) scale</td>
</tr>
<tr>
<td>( 1 &gt; \sin \alpha &gt; 0.2; \text{Red.hoz} 0 )</td>
<td>( V = \text{Di}(\sin \alpha) )</td>
</tr>
<tr>
<td>0.198 &gt; \sin \alpha &gt; 0.02; \text{Red.hoz} 0°</td>
<td></td>
</tr>
</tbody>
</table>

#### Decimal Point

<table>
<thead>
<tr>
<th>( \sin \alpha )</th>
<th>( \alpha )</th>
<th>( \Delta \text{Di} )</th>
<th>( \alpha )</th>
<th>( V )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0447</td>
<td>2° 33.7’</td>
<td>0.001DI</td>
<td>0° 35’</td>
<td>0.01DI</td>
</tr>
<tr>
<td>0.141</td>
<td>8° 06.4’</td>
<td>0.01DI</td>
<td>5° 46.1’</td>
<td>0.1DI</td>
</tr>
<tr>
<td>0.436</td>
<td>25° 50.9’</td>
<td>0.1DI</td>
<td>45°</td>
<td>0.707DI</td>
</tr>
</tbody>
</table>
Stadia/Topographic Slide II Examples

The following examples use the equations given in Table 1. The 10065 manual uses a “digit” counting technique to determine the decimal point, however the decimal point listings in Table 1 are easier to use. The equations listed in Table1 are not given in the manual.

Measured Values
Stadia rod reading , $S = 7 \text{ ft}$; $100S = 700 \text{ ft}$

$\alpha = 25^\circ 50.32'$

**Horizontal Distance, $H = 100S(\cos^2 \alpha) = 567.03 \text{ ft.}$**

a) Place $\cos^2$ index, $0^\circ$, under $100S = 700 \text{ ft}$ on scale A.

b) Move the cursor to $\alpha = 25^\circ 50.32'$ on the $\cos^2$ scale.

c) Read $H = 570 \text{ ft.}$ under the cursor on the A scale. Note; use the decimal point table.

e) Error = 0.53%. Obviously this scale isn’t intended for accuracy.

**Horizontal Distance, $H = 100S - \Delta H$**

$\Delta H = 100S(\sin^2 \alpha) = 132.97 \text{ ft.}$

$H = 700 - 133 = 567.03 \text{ ft.}$

a) Place the cursor over $\alpha = 25^\circ 50.32'$ on the S scale.

b) Read $\sin^2 \alpha = 0.19$ on the A scale. Use the decimal point table.

c) Using the C/D scales calculate $\Delta H = 100S(\sin^2 \alpha) = 133 \text{ ft.}$

d) $H = 100S - \Delta H = 700 - 133 = 567 \text{ ft.}$

e) Error = -0.005%.

**Horizontal Distance, $H = DI - \Delta DI$; DI and $\alpha$ known**

$\Delta Di = Di \sin \alpha (\tan \alpha/2); \quad V = Di \sin \alpha$

$DI = 63 \text{ ft}; \quad \alpha = 26^\circ.$

a) Place the cursor over $\alpha = 26^\circ$ on the S scale

b) Move the C index under the cursor.
c) Move the cursor to DI on the C scale (read V = 27.6 ft under the cursor on the D scale)
d) Move the C index under the cursor
e) Move the cursor to \( \sin \alpha = \sin 26^\circ = 0.438 \) on the Red.hoz.0 scale; labeled \( \tan \alpha/2 \) on the right.

*The two Red. hoz. scales are labeled \( \tan \alpha/2 \), however the numerical values are \( \sin \alpha \). Thus \( \sin \alpha \) on the Red.hoz. scale gives \( \tan \alpha/2 \) on the C scale.*

f) Determine the decimal point; for \( \alpha = 26^\circ \)

\[ \Delta DI > 0.1DI; \ \Delta DI > 6.3 \text{ ft} \]
g) Read \( \Delta DI = DI (\sin \alpha)(\tan \alpha/2) = 6.37 \text{ ft on the D scale.} \)
h) \( H = DI – \Delta DI = 63 \text{ ft.} – 6.37 \text{ ft.} = 56.63 \text{ ft.} \) (theoretical = 56.62402 ft)
i) Error = +0.01%.

**Elevation (Vertical Height), \( V = 100S(\sin \alpha \cos \alpha) = 274.6 \text{ ft.} \)**

a) Place the cursor over \( 100S = 700 \) on the A scale.
b) Bring the \( \cos^2 \) index \( 0^\circ \), on the slide, under the cursor.
c) Move the cursor to \( \alpha = 25^\circ 50.32^\prime \) on the sincos scale. See Table A1 for the decimal point.
d) Read \( V = 100S(\sin \alpha \cos \alpha) = 275 \text{ ft.} \) Note; the Transit height must be subtracted from \( V \).
e) Error = 0.14%.

**Earth Curvature and Refraction correction scale, \( E – R \)**

The elevation, \( V \), for long horizontal distances, 1 km to 10 Km, must be reduced due to the Earths curvature and atmospheric refraction;

\[ E – R = \text{Correction, in meters, } \approx 0.68(H \text{ in Km})^2 \]

The ARISTO System Gruter has the \( E – R \) scale, in meters, on the slide referenced to \( H \) on the right B scale, in Km; under \( H \), in Km, on the right B scale read \( E – R \), in meters, on the \( E – R \) scale. The actual height is less the calculated value, \( V \), by the \( E – R \) correction value.
Slide III
Circular Curves
Circular Curves
(Note; $\Delta$ is our $\alpha$)

R = Radius, $T =$ Sub tangent
$\Delta$ (our $\alpha$) = Tangent Deflection Angle
PI = Point of Tangent Intersection, $L =$ Curve Length
$E =$ External=$PI-A = PI – R$
$C =$ Long Cord = $EC – BC$
$M =$ Mid Ordinate = $A - B$

### Sub tangent $T = R\tan(\alpha/2)$

<table>
<thead>
<tr>
<th>$\alpha$</th>
<th>$R\tan(\alpha/2)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1° 10'</td>
<td>0.0102$R$</td>
</tr>
<tr>
<td>11° 25.27'</td>
<td>0.1$R$</td>
</tr>
<tr>
<td>90°</td>
<td>$R$</td>
</tr>
<tr>
<td>150°</td>
<td>3.73$R$</td>
</tr>
</tbody>
</table>

### Curve Length $L = R(\pi\alpha/180)$

<table>
<thead>
<tr>
<th>$\alpha$</th>
<th>$R(\pi\alpha/180)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1° 40'</td>
<td>0.000106$R$</td>
</tr>
<tr>
<td>5° 07.4'</td>
<td>0.001$R$</td>
</tr>
<tr>
<td>16° 08.3'</td>
<td>0.01$R$</td>
</tr>
<tr>
<td>49° 14.4'</td>
<td>0.1$R$</td>
</tr>
<tr>
<td></td>
<td>$2R\sin(\alpha/2)$</td>
</tr>
</tbody>
</table>

### External $E = R\tan(\alpha/2 \tan(\alpha/4))$

<table>
<thead>
<tr>
<th>$\alpha$</th>
<th>$R\tan(\alpha/2 \tan(\alpha/4))$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1° 40.1'</td>
<td>0.000106$R$</td>
</tr>
<tr>
<td>5° 07.4'</td>
<td>0.001$R$</td>
</tr>
<tr>
<td>16° 08.3'</td>
<td>0.01$R$</td>
</tr>
<tr>
<td>49° 14.4'</td>
<td>0.1$R$</td>
</tr>
</tbody>
</table>

### Long Cord, $C = 2R\sin(\alpha/2)$

<table>
<thead>
<tr>
<th>$\alpha/2$</th>
<th>$2R\sin(\alpha/2)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>3.44'</td>
<td>0.002$R$</td>
</tr>
<tr>
<td>34.38'</td>
<td>0.02$R$</td>
</tr>
<tr>
<td>5° 44.3'</td>
<td>0.2$R$</td>
</tr>
</tbody>
</table>
Circular Curves, Slide III Example

The circular Curves slide is for the professional land surveyor! It’s function is summarized as follows:

This slide is very useful for plotting roads and canals as well as general calculations for circular curves. This slide is also much easier to use in the field than published tables and will give the user substantial time savings on the job.

Preliminary calculations in laying out and designing railway lines and primary highways can easily be performed. Also, since the precision for secondary highways and irrigation canals is less demanding, the slide can be used for these calculations with good results. When using the scales on this slide during calculations, use the index of the scale that contains the value used in the calculation.

Table 2 summarizes the equations for those interested.

Example

\[ R = 500 \text{ ft} \quad \alpha = 10^\circ \]

Sub tangent \( T = R(\tan \alpha/2) = 43.74433 \text{ ft} \)

a) Place the cursor over \( R = 500 \) on the D scale

b) Move the slide; \( \tan \alpha/2 \) index \( (90^\circ) \) under the cursor

c) Move cursor to \( \alpha = 10^\circ \) on the \( \tan \alpha/2 \)

d) Determine the decimal point;

\[ 50 \text{ ft} > R(\tan \alpha/2) > 5 \text{ ft} \]

e) Read \( R = 4303 \text{ ft} \) on the D scale under cursor

f) Error = -0.1%

Curve Length \( L = R(\pi\alpha/180) = 87.26646 \text{ ft} \)

a) Place cursor over \( R = 500 \) on D scale

b) Move slide; \( \tan \alpha/2 \) index, \( 90^\circ \), under cursor

c) Move cursor to \( \alpha = 10^\circ \) on the \( \pi\alpha/180 \) scale

d) Determine the decimal point using Table 2’

\[ R(\pi\alpha/180) \approx 0.2R = 100 \text{ ft} \]

e) Under cursor read \( L = 87.2 \text{ ft} \) on D scale

f) Error \approx +0.008%

External, \( E = \text{External} E = R(\tan \alpha/2)(\tan \alpha/4) = 1.909919 \text{ ft} \)

a) Move slide; C index over \( R = 500 \) on C scale
b) Move cursor to $\alpha = 10^\circ$ on the $\tan(\alpha/2)(\tan\alpha/4)$ scale

c) Determine the decimal point using Table 2;

$$5 \text{ ft} > (\tan\alpha/2)(\tan\alpha/4) > 0.51 \text{ ft}$$

d) Read $E = 1.91$ ft on D scale under cursor

e) Error $\approx +0.004\%$

Long Cord, $C = 2R\sin(\alpha/2) = 87.15574$ ft

a) Move cursor to $\alpha/2 = 5^\circ$ on ST scale

b) Move slide; C index under cursor

c) Move cursor to $2R = 1,000$ on C scale

d) Determine the decimal point using Table 2;

$$0.2R > 2R\sin(\alpha/2) > 0.02R$$

$$100 > 2R\sin(\alpha/2) > 10$$

e) Read $C = 87.25$ under cursor on D scale

f) Error $\approx +0.1\%$