JOS Plus

Basic Slide Rule Operation for Scientists and Engineers

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Scientists and Engineers often must solve problems containing very large and very small numbers; in my case, analog electronic circuit design, frequency ranges from 0 (dc) to in excess of 10 gigahertz (10^9 Hz), current from amperes to nanoamperes (10^{-9}), capacitors from farads to picofarads (10^{-12}), inductors from henries to nanohenries (10^{-9}) and resistors from ohms to megohms (10^6). We usually expressed large and small numbers as; $N.nnnE^{(+/-)x}$, where $E^{(+/-)x} = 10^{(+/-)x}$.

The Oughtred Society's Slide Rule Reference Manual, besides showing a large selection of slide rules (some common and many rare), has an excellent introduction to basic slide rule operation. So why this article? I taught evening classes in solid state circuit design at our local community college from 1967 to 2000, and during the pre calculator days, around 1974, I assumed my students had a firm grasp on slide rule operation. As a slide rule review I handed out a basic overview, much like you see here. This presentation stresses slide rule solutions with large and small numbers, $N.nnnE^{(+/-)x}$, and in many instances takes a different approach to that presented in the Oughtred Manual; this article should be considered as an expansion of the Oughtred Manual and not competition!

Most of my presentation is straightforward and easily committed to memory; who knows when you will be called on to solve a problem during a power outage and find your calculator batteries have died. Note the word "Most" at the beginning of this paragraph. The exceptions are the Log-Log scales ($y = a^N$, $N = \text{Log}_a y$, and $a = \sqrt[N]{y}$); the solution process is simple: however, which Log-Log scale to use can be problematic. The solution to this problem and the use of the other primary slide rule scales, are presented in the following pages.

I

Basic Multiplication, Division, and Decimal Point Determination

I Multiplication; (a)(b) = c

a) Place the C scale index over a on the D scale; under b on the C scale read (a)(b) = c on the D scale.
b) Place b on the CI scale (inverted C scale; 1/b on C = b on CI); under the CI index read (a)(b) = c on the D scale.

II Division, a/b = c

a) Place b on the C scale over a on the D scale; under C index read a/b = c on the D sale.
b) Place the CI index over a on the D scale; Under b the CI index read a/b = c on the D scale.

III Determine the decimal point;

Write all numbers as $N.nnnE^{(+/-)x}$ ($E^{(+/-)x} = 10^{(+/-)x}$) and approximate the solution.

$$(a)(b)/c = d;$$

$$d = (30100)(0.0862)/6020 = (3.01E^{+4})(8.62E^{-2})/6.02E^{+3} \approx (3E^{+4})(10E^{-2})/5E^{+3}$$

$$d \approx 6E^{(+4-2-3)} = 6E^{-1} = 0.6$$

Using the C/D sales;

$$(3.01E^{+4})(8.62E^{-2})/6.02E^{+3} \approx 4.32E^{-1} = 0.432 \quad (0.431)$$

The DF/CF Scales

D/C scales folded at π, some older slide rules folded at √10)

You can multiply and divide on the DF/CF scales just as we do on the D/C scales.

1) $N.nnnE^{(+/-)x}$ on the D scale = $(\pi)N.nnnE^{(+/-)x}$ on the DF scale.
2) $N.nnnE^{(+/-)x}$ on the DF scale = $/N.nnnE^{(+/-)x}/\pi$ on the D scale.

Gauge Marks

Slide rules usually have gauge marks representing various constants; π for example. See the Oughtred Society Slide Rule Reference Manual for a listing of the most common and the Oughtred Society's Pocketbook of the Gauge Marks by Panagiotis Venetsianos for a complete listing.

II

Basic Trigonometry

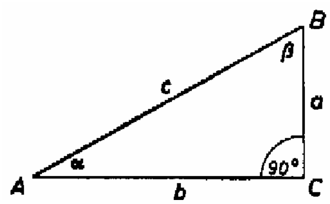

Figure 1.
Triangle with angles and sides labeled

Sinα = a/c; α on the S scale = Sinα on the C scale (C scale limits 0.1 to 1).
$\alpha = Sin^{-1}(a/c)$; a/c on the C scale = α on the S scale.

For small angles

α on the ST/SRT scale = Sinα on the C scale (C scale limits 0.01 to 0.1)

Cosα = b/c; α on the red (Cos) S scale = Cosα on the C scale (C scale limits 0.1 to 1).
$\alpha = Cos^{-1}(b/c)$; b/c on the C scale = α on the red(Cos) S scale.

Tanα = a/b<1; $\alpha<45^o$, α on the black T scale = Tanα on the C scale (C scale limits 0.1 to 1
a/b>1 $\alpha>45^o$, α the red T scale = Tanα on the CI scale (CI scale limits 1 to 10)

$\alpha = Tan^{-1}(a/b)$; a/b<1on the C scale = Tanα ($\alpha<45^o$) on the black T sale.
a/b>1on the CI scale = Tanα ($\alpha>45^o$) on the red T sale.

For small angles

Sinα (in degrees and radians) = Tanα (in degrees and radians) on the ST/SRT scale
(scale limits 0.01 to 0.1)

III

A Scale

x^2; $\sqrt{x}$

Two Decade, 1 to 10 to 100

Note; we can multiply and divide using A/B scales with reduced accuracy

D^2

Note $10^{(+/-)x} = E^{(+/-)x}$

D.ddd$E^{(+/-)x}$ on the D scale = [$(D.ddd)^2$ on the A scale]$E^{(+/-)2x}$; Note the 2x.

$D = 2.682E^{-8}$; $D^2 \approx 7.2E^{-2(8)} = 7.2E^{-16}$ $(7.193E^{-16})$

$D = 4.385E^{4}$; $D^2 \approx 19.2E^{2(4)} = 19.2E^{8}$ $(19.23E^{8})$

$\sqrt{A}$

A on the A scale = $\sqrt{A}$ on the D scale

We must first determine which decade A scale to use; left (1 to 10), or right (10 to 100). Write A as A.aaa$E^{(+/-)x}$. If x is even (0, 2, 4, 6, etc.) use the left A scale, if x is not even, shift the decimal point one place to the right, A = AA.aa $E^{(+/-)x-1}$ and use the right A scale;

a) x is even, use left A scale; $\sqrt{A}$ on the A scale =[($\sqrt{A.aaa}$) on the D scale]$E^{(+/-)x/2}$

Note the x/2

b) x is not even, shift decimal one place to the right, A = AA.aa $E^{(+/-)x-1}$

$\sqrt{A}$ on the A scale =[($\sqrt{AA.aa}$) on the D scale] $E^{[(+/-)x-1]/2}$ and use the right A scale.

Note the (x-1)/2

We can easily tell if a shift in decimal point is necessary.

1) $A = 3.725E^{+5}$. x = +5, is not even so shift the decimal one place to the right, $A = 37.25E^{+5-1} = 37.25E^{+4}$ (right A scale). $\sqrt{A}$ = [($\sqrt{37.25}$) on the D scale]$E^{+4/2}$ $\approx$ $6.1E^{2}$ $(6.103E^{2})$.
2) $A = 6.817E^{-13}$. x = -13 is not even so again we must shift the decimal point one place to the right; $A = 68.17E^{-13-1} = 68.17^{-14}$ (right A scale); $\sqrt{A}$ = [($\sqrt{68.17}$) on the D scale]$E^{-14/2}$ $\approx$ $8.26E^{-7}$ $(8.256E^{-7})$.

We will now write A by inspection

3) $A = 5.542E^{+6}$ (left A scale); $\sqrt{A} \approx 2.36E^{3}$ $(2.354E^{3})$.
4) $A = 8.249E^{-17} = 82.49E^{-18}$ (right A scale); $\sqrt{A} \approx 9.07E^{-9}$ $(9.082E^{-9})$.
5) $A = 2.262E^{-5} = 22.62E^{-6}$ (right A scale); $\sqrt{A} \approx 4.75E^{-3}$ $(4.756E^{-3})$.
6) $A = 4.545E^{-12}$ (left A scale); $\sqrt{A} \approx 2.13E^{-6}$ $(2.132E^{-6})$.
7) $A = 3.258E^{+11} = 32.58E^{+10}$ (right A scale); $\sqrt{A} \approx 5.71E^{+5}$ $(5.708E^{+5})$.

IV

K Scale

x^3; $^3\sqrt{x}$

Three Decades; 1 to 10 to 100 to 1,000

D^3

Note $10^{(+/-)x} = E^{(+/-)x}$

D.ddd$E^{(+/-)x}$ on the D scale = [$(D.ddd)^3$ on the K scale]$E^{(+/-)3x}$; Note the 3x.

$D = 4.879E^{+4}$; $D^3 \approx 116E^{+3(4)} = 116E^{+12}$ ($116.14E^{+12}$)

$D = 2.385E^{-3}$; $D^3 \approx 13.6E^{-3(3)} = 13.6E^{-9}$ ($13.566E^{-9}$)

$D = 1.738E^{-4}$; $D^3 \approx 5.23E^{-3(4)} = 5.23E^{-12}$ ($5.2499E^{-12}$)

$^3\sqrt{K}$

K on the proper K scale = $^3\sqrt{K}$ on the D scale

We must determine which K decade scale to use. K must be K.kkk$E^{(+/-)x}$ for the left K scale, KK.kk$E^{(+/-)x}$ for the center K scale, or KKK.k$E^{(+/-)x}$ for the right K scale, where x must be a multiple of 3; 0, 3, 6, 9, etc.

Write K = K.kkk$E^{(+/-)x}$

1) If x is a multiple of 3, no problem; use the left K scale

   $^3\sqrt{K}$ = [($^3\sqrt{K.kkk}$) on the D scale]$E^{[(+/-)x]/3}$; Note the x/3.

2) If x isn't a multiple of 3, move the decimal point one place to the right;

   K =KK.kk$E^{(+/-)x-1}$

   if (+/-x) – 1 is a multiple of 3, again no problem; use the center K scale.

   $^3\sqrt{K}$ = [($^3\sqrt{KK.kk}$) on the D scale]$E^{[(+/-)x-1]/3}$; Note the [(x)-1]/3

3) If [(+/-x)-2] still is not a multiple of 3, move the decimal point one more time to the right; and use the right K scale; K =KKK.k$E^{(+/-)x-2}$,

   $^3\sqrt{K}$ = [($^3\sqrt{KKK.k}$) on the D scale]$E^{[(+/-)x-2]/3}$

   Note the [(x-2]/3]

In practice it is easy to find $^3\sqrt{K}$ as shown below;

1) $K = 4.658E^{-5}$. The exponent, -5, isn't a multiple of 3 so shift the decimal one place to the right; $K = 46.58E^{-6}$. The exponent is a multiple of 3, thus

   $^3\sqrt{K}$ = [($^3\sqrt{46.58}$) on the D scale]$E^{-6/3} \approx 3.6E^{-2}$ ($3.598E^{-2}$)

2) $K = 6.592E^{-7}$. The exponent, -7, is not a multiple of 3 so shift the decimal one place to the right, $K = 65.92E^{-8}$. The exponent still is not a multiple of 3, so shift the decimal to the right one more time; $K = 659.2E^{-9}$ and the exponent is a multiple of 3, use the right K scale.

   $^3\sqrt{K}$ = [($^3\sqrt{659.2}$) on the D scale]$E^{-9/3} \approx 8.7E^{-3}$ ($8.703E^{-3}$)

3) $K = 7.385E^{+5} = 738.5E^{+3}$ (right K scale);

   $^3\sqrt{K}$ = [($^3\sqrt{738.5}$) on the D scale]$E^{+3/3} \approx 90.5$ (90.39)

4) $K = 8.752E^{+13} = 87.52E^{+12}$ (center K scale);

$^3\sqrt{K} = [(^3\sqrt{87.52}) \text{ on the D scale}]E^{+12/3} \approx 4.44E^{+4}$ $(4.4399E^{+4})$

V

The Log L scale

$N = Log_{10}y$

$N = Log_{10}$ y Note; $E^x = 10^x$ Write y as; $y = Y.yyyE^C$ where Y.yyy>0 $N = Log_{10}y = Log_{10}(Y.yyyE^C) = Log_{10}(E^C) + Log_{10}(Y.yyy)$ = C (characteristic)+ ($Log_{10}(Y.yyy)$ = 0.mmm mantissa) 1) place cursor over y = Y.yyy on the D scale 2) read the mantissa ($Log_{10}(Y.yyy)$ = 0.mmm) below cursor on the L scale 3) N = characteristic (C) + mantissa ($Log_{10}(Y.yyy)$ = 0.mmm)
y>1; N>0 (positive) Let y = 2,500 = $2.5E^3$ (characteristic C =3) $N = Log_{10}y = Log_{10}(Y.yyyE^C) = Log_{10}(2.5E^3) = Log_{10}(E^3) + Log_{10}(2.5)$ = 3 (characteristic) + 0.39794 (mantissa) = 3.39794 1) place cursor over y =Y.yyy= 2.5 on the D scale 2) read the mantissa, ($Log_{10}(Y.yyy) = Log_{10}(2.5)$ = 0.mmm ≈ 0.398) below cursor on the L scale 3) N = characteristic (C = 3) + (mantissa ($Log_{10}(Y.yyy)$ = 0.mmm ≈ 0.398) ≈ 3 + 0.398 = 3.398 (3.39794)
0<y<1; N<0 (negative) Let y = 0.0025 = $2.5E^{-3}$ (characteristic C = -3) $N = Log_{10}y = Log_{10}(2.5E^{-3}) = Log_{10}(E^{-3}) + Log_{10}(2.5)$ = -3 (characteristic) + 0.39794 (mantissa) = -2.60206 1) place cursor over y = Y.yyy = 2.5 on the D scale 2) read the mantissa, $Log_{10}(Y.yyy) = Log_{10}(2.5)$ = 0.mmm ≈ 0.398) below cursor on the L scale 3) N = characteristic (C = -3) + mantissa ($Log_{10}(Y.yyy)$ = 0.mmm) ≈ 0.398) ≈ -3 + 0.398 ≈ -2.602 (-2.60206) Note; Log Tables give N as [(10 + C)+ mantissa (0.mmm)] - 10. For our example, C = -3 N = [(10 - 3) + 0.39794 (mantissa)] - 10 = [7 + 0.39794] – 10 = 7.39794 - 10 = (–2.60206)
$N = Log_a y$ $N = Log_a y = (Log_{10}y)/ (Log_{10}a)$ This is the method for finding N using a calculator

VI

The L Scale

$y = 10^{N}$

$y=10^{N}$

Note; $E^{x} = 10^{x}$

Write N as; C.mmm, where C is the characteristic and 0.mmm is the mantissa

$y = 10^{N} = [10^{C\ (\text{characteristic})}][10^{(0.\text{mmm}\ (\text{mantissa})}] = [10^{0.\text{mmm}}][10^{C}] = Y.yyyE^{C}$

1) place cursor over the mantissa (0.mmm) on L scale
2) read ($10^{0..\text{mmm}\ (\text{mantissa})}$ = Y.yyy) below the cursor on the D scale
3) $y =10^{N} = 10^{0.\text{mmm}\ (\text{mantissa})}]\ [10^{C\ (\text{characteristic})}] = 10^{0.\text{mmm}}[10^{C}] = Y.yyyE^{C}$

y>1; N>0 (positive)

Let N = 3.39794 = 3(characteristic, C) + 0.39794(mantissa, 0.mmm)

1) place cursor over the mantissa (0.mmm) = 0.39794 on L scale
2) read ($10^{0..\text{mmm}\ (\text{mantissa})}$ = Y.yyy) ≈ 2.5 below the cursor on the D scale
3) $y =10^{N} = 10^{0..\text{mmm}}$ (mantissa)] $[10^{C}$ (characteristic)] $= 10^{(0.39794)}[10^{3}] \approx 2.5E^{3}$ $(2.5000E^{3})$

0<y<1; N<0 (negative)

Write N as; N = -N.nnn = -N - 0.nnn and we must convert –N – 0.nnn to;
characteristic (C) + mantissa (0.mmm) as shown below.
characteristic (C) = N-1
mantissa (0.mmm) = 1 – 0.nnn

Thus $y = 10^{N} = 10^{-\ N.\text{nnn}} = [10^{\text{characteristic } C}][10^{\text{mantissa } (0.\text{mmm})}] = [10^{(N-1)}][10^{(1-0.\text{nnn})}]$

Or; $y = [10^{0.\text{mmm})}]\ [10^{\ C}]$

Let N = -2.60206 = -N - 0.nnn = -2 – 0.60206
characteristic (C) = N-1 = -2-1 = -3
mantissa (0.mmm) = 1 - 0.nnn = 1 - 0.60206 = 0.39794

$y = 10^{0.39794}E^{-3} = 2.500E^{-3}$

1) place cursor over the mantissa (0.mmm) = 0.39794 on L scale
2) read ($10^{0.\text{mmm}\ (\text{mantissa})}$ = Y.yyy) ≈ 2.5 below the cursor on the D scale
3) $y =10^{N} = 10^{0.\text{mmm}}$ (mantissa)] $[10^{C}$ (characteristic)] $= 10^{(0.39794)}][10^{-3}] \approx 2.5E^{-3}$ $(2.5000E^{-3})$

$y = a^{N} = 10^{N\log a}$

where $N\log a = N\log_{10}a$

The Log Log scales solve this example quite easily.

VII

The Log-Log Scales
Log Log Scale Nomenclature, Scale Limits and Operation
$N = Lny = Log_e y$; $y = e^{+/-}N$

Log Log Scale Nomenclature and Limits
(inverted, LL0, scales in red)
Note; LL/LLO are mated scales; Z on LLZ scale = 1/Z on LL0Z scale; 1/Z on LL0Z scale = Z on LLZ scale

K&E 4080/81/83	K&E Deci-Lon	Nestler 0292	Post 1461	Chinese 1003	Scale Limits e^N	N limits for the D scale
LL3	Ln3	LL3	LL3	ln_1	$e^1 \rightarrow e^{10}$	1<N<10
LL03	Ln-3	LL_03	LL/3	$\mathit{ln_1}$	$e^{-1} \leftarrow e^{-10}$	-1<N<-10
LL2	Ln2	LL2	LL2	ln_0	$e^{0.1} \rightarrow e^1$	0.1<N<1
LL02	Ln-2	LL_02	LL/2	$\mathit{ln_0}$	$e^{-0.1} \leftarrow e^{-1}$	-0.1<N<-1
LL1	Ln1	LL1	LL1	ln_{-1}	$e^{0.01} \rightarrow e^{0.1}$	0.01<N<0.1
LL01	Ln-1	LL_01	LL/1	$\mathit{ln_{-1}}$	$e^{-0.01} \leftarrow e^{-0.1}$	-0.01<N<-0.1
	Ln0	LL0	LL0	ln_{-2}	$e^{0.001} \rightarrow e^{0.01}$	0.001<N<0.01
	Ln-0	LL_00	LL/0	$\mathit{ln_{-2}}$	$e^{-0.001} \leftarrow e^{--.01}$	-0.001<N<-0.01

N = Ln y

y>1 (on LLy scale) ; N>0 positive (on D scale)

1) place cursor over y on LLy scale
2) under cursor read +N = Lny on D scale
3) decimal point same as +N (N>0) scale limits for the LLy scale number

Let y = 1.35 (LL2 scale); D scale limits 0.1<N<1
$N = Ln(1.35) \approx 0.3\ (0.30010)$

0<y<1 (on LL0y scale); N<0 negative (on D scale)

1) place cursor over y on LL0y scale
2) under cursor read –N = Lny on D scale
3) decimal point same as -N (N scale limits for the LL0y scale number)

Let y = 0.74074 (LL02 scale); D scale limits -0.1<N<-1
$N = Ln(0.74074) \approx -0.3\ (-0.30011)$

$y = e^{+/-N}$

y>1 (on LLy scale); N>0 positive (on D scale)

1) place cursor over N on D sale
2) under cursor read $y = e^{+N}$ on LLy scale with +N scale limits

Let N = 0.30011; D scale limits 0.1<N<1 (LL2 scale)
$y\ (LL2) = e^{0.30011} \approx 1.35\ (1.3499)$

0<y<1 (on LL0y scale); N<0 negative (on D scale)

1) place cursor over N on D sale
2) under cursor read $y = e^{-N}$ on LL0y scale with N scale limits

Let N = -0.30011; D limits -0.1<N<-1 (LL02 scale)
$y\ (LL02) = e^{-0.30011} \approx 0.7405\ (0.74074)$

VIII

Log-Log Scale Summary

The LL/LL0 scales are mated; x the LLx scale = 1/x on the LL0x scale, and vice versa

The LL/LL0 scales as a function of the quadrant

II	I
$y>1$; $N<0$ (negative); $0<a<1$ $y=a^{-N} = 1/ a^{N}$ $N = \log_{a}y$ $a= y^{-1/N} = 1/ y^{1/N} = 1/ (\sqrt[N]{y})$ y on LLy scale -N on C scale a on L0a scale	$y>1$; $N>0$ (positive); $a>1$ $y =a^{N}$ $N = \log_{a}y$ $a = y^{1/N} = \sqrt[N]{y}$ y on LLy scale +N on C scale a on LLa scale
III $0<y<1$; $N<0$ (negative); $a>1$ $y = a^{-N} = 1/ a^{N}$ $N = \log_{a}y$ $a = y^{-1/N} = 1/(y^{1/N})$ y on LL0y scale -N on C scale a on LLa scale	**IV** $0<y<1$; $N>0$ (positive); $0<a<1$ $y = a^{N}$; $N = \log_{a}y$ $a = y^{1/N} = \sqrt[N]{y}$ y on LL0y scale +N on C scale a on LL0a scale

IX

Log-Log Scales

Power

$y = a^{+/-N}$; $y = a^{(-N)} = 1/a^N$

The LL and LL0 scales mated; Z on LL = 1/Z on LL0, Z on LL0 = 1/Z on LL

The solution scales depend on which C scale index, left or right, is used

$y = a^N$; $y = a^{(-N)} = 1/a^N$ 1) move cursor to a on LLa/LL0a scale 2) move slide; C index under cursor 3) move cursor to N on C scale 4) determine the proper LLy/LL0y scale as discussed below 5) read $y = a^N$ under cursor on proper LLy/LL0y scale	
LL/LL0 Solution Scales a>1 N>0; a on LLa scale, read y on LLy scale N<0; a on LLa scale, read y on LL0y scale 0<a<1 N>0; a on LL0a scale, read y on LL0y scale N<0; a on LL0a scale, read y on LLy scale	
LLa/LL0a solution scales Write N as $N.nnnE^{(+/-)x}$ ($E^{(+/-)x} = 10^{(+/-)x}$) $N = 51.3 = 5.13E^1$, x = 1; $N = 0.00329 = 3.29E^{-3}$, x = -3	
Left C index over a on LLa/LL0a scale **knowing $NE^{(+/-)x}$** **LLy/LL0y solution scale = LLa/LL0a scale + x**	**Right C index over a on LLa/LL0a scale** **knowing $NE^{(+/)-x}$** **LLy/LL0y solution scale = LLa/LL0a scale + x + 1**
$y = a^N$ **a>1 on LLa scale; N>0 (positive) on C scale; y>1 on LLy scale**	
Let a = 1.02 (LL1 scale) and $N = 1.5124E^2$; x = 2 LLy scale = LLa scale + 1 = 1 + 2 = 3 (LL3 scale) y (LL3) = $(1.02)^{(1.5124E2)} \approx 20$ (20.000)	Let a = 7 (LL3 scale) and $N = 7.1447E^{-3}$; x = -3 LLy scale = LLa scale + x + 1 = 3 - 3+1 = 1 (LL1 scale) y (LL1) = $7^{(7.1447E-3)} \approx 1.014$ (1.0140)
$a = y^{(-N)} = 1/y^{(N)}$ **a>1 on LLa scale; N<0 (negative) on C scale; 0<y<1 on LL0y scale**	
Let a = 1.02 (LL1 scale) and $N = -1.5124E^2$; x = 2 LL0y scale = LLa scale + 1 = 1 + 2 = 3 (LL03 scale) y (LL_o3) = $(1.02)^{(-1.5124E2)} \approx 0.05$ (0.05000)	Let a = 7 (LL3 scale) and $N = -7.1447E^{-3}$; x = -3 LL0y scale = LLa scale + x + 1 = 3 – 3 + 1 = 1 (LL1 scale) y(LL01) = $7^{(-7.1447E-3)} \approx 0.9862$ (0.98619)
$y = a^N$ **0<a<1 on LL0a scale; N>0 (positive) on C scale; 0<y<1 on LL0y scale**	
Let a = 0.9804 (LL01 scale) and $N = 1.5124E^2$; x = 2 LL0y scale = LL0a scale + 1 = 1 +2 = 3 (LL03 scale) y (LL03) = $(0.9804)^{(1.5124E2)} \approx 0.05$ (0.0500)	Let a = 0.14286 (LL03 scale) and $N = 7.1447E^{-3}$ x = -3 LL0y scale = LL0a scale + x +1 = 3 – 3 + 1 = 1 (LL01 scale) y (LL01) = $(0.14286)^{(7.1447E-3)} \approx 0.9862$ (0.98619))
$= y^{-N} = 1/a^N$ **0<a<1 on LL0a scale; N<0 (negative) on C scale; y>1 on LLy scale**	
Let a=0.9804 (LL01 scale) and $N=-1.5124E^2$; x = 2 LLy scale = LL0a scale + 1 = 1 + 2 = 3 (LL3 scale) y (LL3) = $(0.9804)^{(-1.5124E2)} \approx 20$ (20.000)	Let a = 0.14286 (LL03 scale) and $N = -7.1447E^{-3}$; x = -3 LLy scale = LL0a scale +x +1=3 – 3 +1 = 1 (LL1 scale) y (LL1) = $(0.14286)^{(-7.1447E-3)} \approx 1.014$ (1.0140))

X

The Log-Log Scales
Roots
$a = y^{(1/N)} = {}^{N}\sqrt{y}$; $a = y^{(1/-N)} = y^{-(1/N)} = 1/{}^{N}\sqrt{y}$
The LL and LL0 scales mated; Z on LL = 1/Z on LL0, Z on LL0 = 1/Z on LL

The solution scales depend on which C scale index, left or right, is used

$a = y^{1/N} = {}^{N}\sqrt{y}$; $a = y^{(1/-N)} = 1/{}^{N}\sqrt{y}$ 1) move cursor to y on LLy/LL0y scale 2) move slide; N on C scale under cursor 3) move cursor to C index 4) determine the proper LLa/LL0a scale as described below 5) read $a = y^{1/N} = {}^{N}\sqrt{y}$ under cursor on proper LLa/LL0a scale	
LL/LL0 Solution Scales y>1 N>0; y on LLy scale, read a on LLa scale N<0; y on LLy scale, read a on LL0a scale 0<y<1 N>0; y on LL0y scale, read a on LL0a scale N<0; y on LL0y scale, read a on LLa scale	
LLa/LL0a solution scales Write N as $N.nnnE^{(+/-)x}$ ($E^{(+/-)x} = 10^{(+/-)x}$) $N = 51.3 = 5.13E^{1}$, x = 1; $N = 0.00329 = 3.29E^{-3}$, x = -3	
Left C index over a on LLa/LL0a scale **knowing $N^{(+/-)x}$** **LLa/LL_0a solution scale = LLy/LL_0y scale - x**	**Right C index over a on LLa/LL_0a scale** **knowing $N^{(+/-)x}$** **LLa/LL0a solution scale = LLy/LL_0y scale - x - 1**
$a = y^{(1/N)} = {}^{N}\sqrt{y}$ **y>1 on LLy scale; N>0 (positive) on C scale; a>1 on LLa scale**	
let y = 20 (LL3 scale); $N = 1.5128E^{2}$, x =2 LLa scale = LLy scale – x = 3 – 2 = 1 (LL1 scale) a (LL1) = $y^{(1/N)}$ = $(20)^{(1/1.5128E2)} \approx 1.02$ (1.02000)	let y = 1.0114 (LL1 scale); $N = 7.1447E^{-3}$, x = -3 LLa scale = LLy scale – x – 1= 1 - (-3) - 1 = 3 (LL3 scale) a (LL3) = $y^{(1/N)}$ = $(1.01140)^{(1/7.1447E-3)} \approx 4.89$ (4.88619)
$a = y^{-(1/N)} = 1/y^{(1/N)} = 1/{}^{N}\sqrt{y}$ **y>1 on LLy scale; N<0 (negative) on C scale; 0<a<1 on LL0a scale**	
let y = 20 (LL3 scale); $N = -1.5128E^{2}$, x=2 LL0a scale = LLy scale – x = 3 – 2 = 1 (LL01 scale) a (LL01) = $y^{-(1/N)}$ = $(20)^{-(1.5128E2)} \approx 0.9804$ (0.98039)	let y = 1.0114 (LL1 scale); $N = -7.1447E^{-3}$, x = -3 LL0a scale = LL0y scale – x – 1 = 1 - (-3) -1 = 3 (LL03 scale) a (LL03) = $y^{-(1/N)}$ = $(1.0114)^{-(1/7.1447E-3)} \approx 0.2045$ (0.20463)
$a = y^{(1/N)} = {}^{N}\sqrt{y}$ **0<y<1 on LL0y scale ; N>0 (positive) on C scale; 0<a<1 on LL0a scale**	
let y = 0.05000 (LL03 scale); $N = 1.5128E^{2}$, x=2 LL0a scale = LLy scale – x = 3 – 2 = 1 (LL01 scale) a (LL01) = $y^{(1/N)}$ = $(0.050)^{(1/1.5128E2)} \approx 0.9804$ (0.98039)	let y = 0.98873 (LL01 scale); $N = 7.1447E^{-3}$, x = -3 LL0a scale = LL0y scale – x – 1 = 1 - (-3) -1 = 3 (LL03 scale) a (LL03) = $y^{(1/N)}$ = $(0.98873)^{(1/7.1447E-3)} \approx 0.2045$ (0.20467)
$a = y^{-(1/N)} = 1/y^{(1/N)} = 1/{}^{N}\sqrt{y}$ **0<y<1 on LL0y scale ; N<0 (negative) on C scale; a>1 on LLa scale**	
let y = 0.05000 (LL03 scale); $N = -1.5128E^{2}$, x=2 LLa scale = LL0y scale – x = 3 – 2 = 1 (LL1 scale) a (LL1) = $y^{-(1/N)}$ = $(0.050)^{-(1/1.5128E2)} \approx 1.02$ (1.02000)	let y = 0.98873 (LL01 scale); $N = -7.1447E^{-3}$, x = -3 LLa scale = LL0y scale – x – 1 = 1 - (-3) -1 = 3 (LL3 scale) a (LL3) = $y^{-(1/N)}$ = $(0.98873)^{-(1/1.5128E2)} \approx 4.89$ (4.88619)

XI

The Log-Log Scales
$N=Log_a y$

The solution scales depend on which C scale index, left or right, is used

$N = Log_a y$

1) move cursor to a on LLa/LL0a scale
2) move slide; C index under cursor
3) move cursor to y on LLy/LL0y scale
4) determine $NE^{(+/-)x}$, N positive or negative, and exponent, (+/-)x , as described below
5) read $N = Log_a y$ under cursor on C scale

LLa/LL0a and LLa/LL0y scales
a>1
y>1; a on LLa scale, y on LLy scale, read positive N on C scale
0<y<1; a on LLa scale, y on LL0y scale, read negative N on C scale
0<a<1
y>1; a on LL0a scale, y on LLy scale, read negative N on C scale
0<y<1; a on LL0a scale, y on LL0y scale, read positive N on C scale

NE^x on the C scale
Note $E^{(+/-)x} = 10^{(+/-)x}$

Left C index over a on LLa/LL0a scale **x = LLy/LL0y scale - LLa/LL0a scale**	**Right C index over a on LLa/LL0a scale** **x = LLy/LL0y scale - LLa/LL0a scale - 1**
a >1 on LLa scale; y>1 on LLy scale; N>0 (positive) on C scale	
Let a = 1.02 (LL1 scale) and y = 20 (LL3 scale) x = LLy scale - LLa scale = 3 - 1= 2; NE^2 $N = Log_{(1.02)}(20) \approx 1.515E^2$ ($15128E^2$)	Let a = 7 (LL3 scale) and y = 1.014 (LL1 scale) x = LLy scale - LLa scale - 1 = 1 – 3 – 1 = -3; NE^{-3} $N = Log_7(1.014) \approx 7.15E^{-3}$ ($7.1447E^{-3}$)
a >1 on LLa scale; *0<y<1 on LL0y scale; N<0 (negative) on C scale*	
Let a = 1.02 (LL1 scale) and y = 0.05 (LL03 scale) x = LL0y scale - LLa scale = 3 – 1 = 2; NE^2 $N = Log_{(1.02)}(0.05) \approx -1.515E^2$ ($-15128E^2$)	Let a = 7 (LL3 scale) and y = 0.98619 (LL01 scale) x = LL0y scale - LLa scale - 1 = 1 – 3 – 1 = -3; NE^{-3} $N = Log_7(0.98619) \approx -7.15E^{-3}$ ($-7.1447E^{-3}$)
0<a<1 on LL0a scale; *y>1 on LLy scale* ; *N<0 (negative) on C scale*	
Let a = 0.9804 (LL01 scale) and y = 20 (LL3 scale) x = LLy scale – LL0a scale = 3 – 1 = 2; NE^2 $N = Log_{(0.9804)}(20) \approx -1.515E^2$ ($-15128E^2$)	Let a = 0.14286 (LL03 scale) and y = 1.014 (LL1 scale) x = LLy scale – LL0a scale –1 = 1 - 3 – 1 = -3; NE^{-3} $N = Log_{(0.14286)}(1.014) \approx -7.15E^{-3}$ ($-7.1447E^{-3}$)
0<a<1 on LL0a scale; *0<y<1 on LL0y scale; N*>0 (positive) *on C scale*	
Let a = 0.9804 (LL01 scale) and y = 0.05 (LL03 scale) x = LL0y scale – LL0a scale = 3 – 1 = $2NE^2$ $N = Log_{(0.9804)}(0.05) \approx 1.515E^2$ ($1.5128E^2$)	Let a = 0.14286 (LL03 scale) and y = 0.98619 (LL01 scale) x = LL0y scale – LL0a scale – 1 = 1 – 3 – 1 = -3; NE^{-3} $N = Log_{(0.14286)}(0.98619) \approx 7.15E^{-3}$ ($7.1447E^{-3}$)