

Nikolaus Goldmann's architectural rods

Extended Version

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Architecture and mathematics

For centuries, architecture was both art and science. The only antique treatise on architecture that has survived was Vitruvius' *De architectura* (c. 40 B.C.) [15]. MARCUS VITRUVIUS POLLIO was a Roman architect and engineer for the Emperor AUGUST. His work was the most important source for scholars. Vitruvius emphasised the importance of geometry and arithmetic, especially proportions. During the Renaissance Vitruvius was rediscovered, and architecture became a part of mathematics. As a result, e.g. the *Anfangsgründe aller mathematischen Wissenschaften* (1710) from CHRISTIAN WOLFF (1679–1754) dealt with architecture in an axiomatic way [18]. As customary during that time, it distinguished between civil and military architecture. An attractive part of civil architecture was the *theory of columns*. At the same time architecture was presented as an extensive theory by NIKOLAUS GOLDMANN (1611–1665) in his books on civil and military architecture that were edited by LEONHARD CHRISTOPH STURM (1669–1719). Both architects had a background in mathematics.

Architects needed *mathematical instruments* for their constructions, e.g. dividers, callipers, rulers, protractors, and proportional compasses [7]. During the seventeenth and eighteenth century, the elements of architecture were part of the university lectures on mathematics. The students gained theoretical knowledge by the lectures and practical experience by the use of instruments. The University of Würzburg had compiled a collection of mathematical instruments in the *Mathematical cabinet* of the philosophical faculty. An inventory of this collection from 1707 still exists in the *Bibliotheca Apostolica Vaticana* in Rome and gives an interesting impression of the collection at the beginning of the 18th century [9].

The historical cabinet with its voluminous collection does not exist any more. Most of its items were sold in 1877 to the *Bayerisches Nationalmuseum* (BN) in München [16]. The most precious piece was the *Planetenmaschine* made by JOHANN GEORG NESSTFELL (1694–1762). Some more instruments were given on loan to the *Deutsches Museum München* in 1905 und 1910. Most of the Würzburg instruments in the BN remained unnoticed. Therefore, in 2004 I started studying the instruments and compared the present inventory with the inventory from 1707.

Nikolaus Goldmann's architectural rods

Reading the inventory of the BN I discovered a set of six brass architectural rods in a leather-covered cylindrical case with a slip top of unknown origin. (Fig. 1). The rods have a three-sided prismatic shaft of 23.4 cm and a grip of 3 cm. The sides have a width of 1.3 cm. There are five rods with the signatures: TVS:, DOR:, ION:, COR:, ROM. A sixth rod has the letters C. and V. on one side, T. and D. at the other, and I. and R. at the third

one. There are two scales with numbers on each side of the rods. The abbreviations give hints to the classic orders of column: *Tuscan* (TVS: resp. T.), *Doric* (DOR: resp. D.), *Ionic* (ION: resp. I.), *Corinthian* (COR: resp. C.), and *Roman* (ROM: resp. R.).



FIGURE 1.

(a) Prismatic Rods with case and top; (b) part of the Tuscan rod
Bayerisches Nationalmuseum München (Inv. Nr. Phys 176-182).

The inventory of the Würzburg collection in the Vatican from 1707 lists a suitable instrument under the title “metallina“ as follows:

“*Columnae architectonicae cum accuratissimis divisionibus, in vagina sexagona intus, for(r)is rotundata.*“ ([9], p. 180)

This entry, most likely, refers to the instrument of the BN.

In a book on historical drawing instruments I found a picture of the same instrument with the hint that the photo shows the architectural rods of Nikolaus Goldmann from the *Whipple Museum of the History of Science* in Cambridge ([7], p. 142). In addition, the *Tractatus de stylometris* of Nikolaus Goldmann appeared in 1662 at Leiden. The title (Fig. 2) shows the six rods, and they are called building rods (*stylometra*). This book is a handbook for the use of the building-rods. I was very happy to find a copy of that book in our University Library. The book has the handwritten notice: “*Facultatis Philosoph. Wirceb. 1754*“ that refers to the Philosophical Faculty of Würzburg. One can assume that the book and the instrument originally belonged together. According to the book, Nikolaus Goldmann was the inventor of the architectural rods.

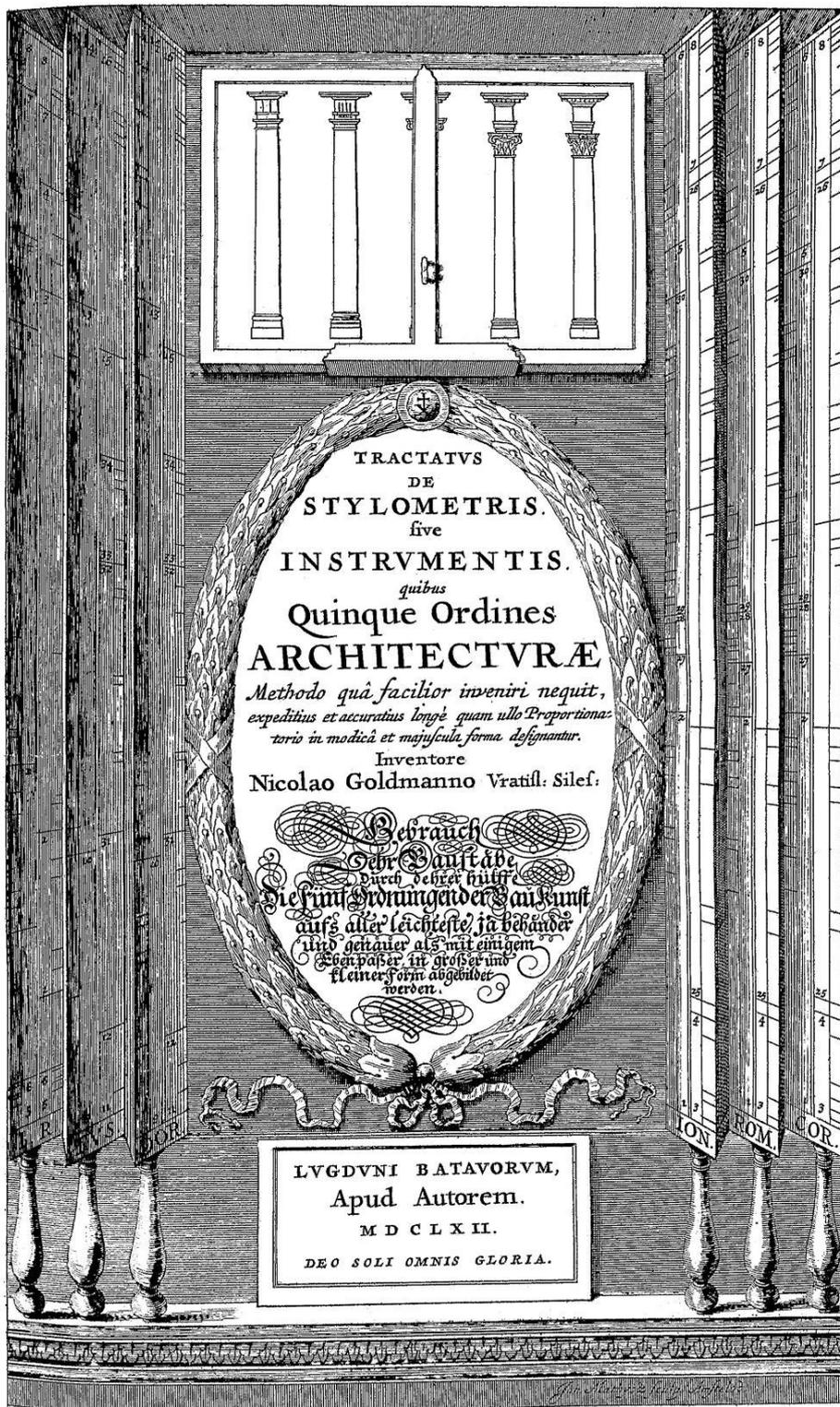


FIGURE 2.

Title of Goldmann's book about his architectural rods.
University Library Würzburg (35/A 11.32)

Nikolaus Goldmann

Nikolaus Goldmann was a mathematician who had been working in Leiden [12]. With his *Elementa architecturae militaris* [2], he gave a mathematically founded introduction into military architecture. Through his contributions to the theory of columns, he earned reputation as a theorist in architecture. His construction of the *Ionic volute* (spiral) convinced by its simplicity and became customary e. g. ([1]). A treatise on civil architecture was only edited in 1696 by Leonhard Christoph Sturm as *Vollständige Anweisung zu der Civil-Bau-Kunst* at Wolfenbüttel [5]. It became a classic [12].

Goldmann called himself a *Vratislaviensis Silesius* (Silesian from Breslau) on the title pages of his books, because he was baptized in Breslau on September 29th, 1611. In 1629 he began to study law at Leipzig and moved to Leiden in 1632 where he studied law and mathematics. Finally, he worked as private scholar and teacher without belonging to the staff of the university [10]. He remained there until the end of life in the first half of the year 1665 ([13], p. 60).

Goldman experimented with his architectural rods several years until 1662, when they were mature. He wrote:

„Wie viel Jahre ich über dieser erfindung geschwitzet / ist besser das ich schweige / als vor einen aufschneider gehalten werde. Vor etlichen Jahren habe ich Baustäbe / etlichen die bey mir die Baukunst gelehret / geoffenbahret / dieselbe werf ich als unzeitige geburthen hinweg: ahngesehn daß ich auf diese sechs Baustäbe soviel als vor auf zehne / gebracht habe. Jehne wahren mein Lehrstücke / diese sollen das Meisterstücke sein / ahngesehen sie nuhmehr also verbessert / ausgearbeitet / zuesammen gezogen / und aufgehhalten sein / daß sie mich endlich vergnügen.“ ([4], S. 3)

He first used wooden rods with printed-paper scales on them. However, they were too inaccurate. Therefore, he chose for brass. He would have preferred silver, but he abandoned the idea because of the high price of silver. He wrote:

„Bishero sein diese Baustäbe von messing gut gefunden: silber wehre besser / weil auf dehr weißlichten fläche / die schwarze strichlein dehr einkerbungen besser inns auge fielen. Man saget es würden teure Kunstzeuge werden; derogleichen geitzhalse rahte ich / daß er nicht so viel räusche sauffe / und das geld das er gotlose versäuft zum silber erspahre.“ ([4], S. 2) In 1662, Goldmann had edited and published on his own the *Tractatus de stylometris* at Leiden (Lugdunum Batavorum) [4]. He dedicated it to prince elector FRIEDRICH WILHELM of Brandenburg and to his governor prince JOHANN MORITZ of Sachsen, because he hoped to get financial support, which he finally received [12]. In addition to serving as a handbook for the architectural rods, it is an introduction to the theory of columns that follows Marcus Vitruvius Pollio and GIACOMO BAROZZI DA VIGNOLA (1507–1573), the Italian theorist of architecture [14]. A detailed description of Goldmann's life and work was given by Jeroen Goudeau [6].

Johann Eggerich Frerss

Goldmann refers to Iohannes Eccericus Freerius as the maker of the architectural rods. He was a mechanic in Leiden. In his *Tractatus de stylometris* he noted, that one could order

the rods for “*13 Imperialium pretio*“, which was about 2 ½ Gulden [8]. His German name was JOHANN EGGERICH FRERSS. On April 18th, 1664, the prince elector FRIEDRICH WILHELM appointed him as court mechanic at Cölln by the Spree River [12]. Since 1709, Cölln by the Spree is part of Berlin. Some of his instruments can be seen in Museums at Bern, Dresden, Hamburg, and Kassel. They are all signed. Unfortunately, the architectural rods are unsigned, but Goldmann’s reference to Frerss leads us to the assumption that Frerss was the manufacturer [17].

The classical Columns

The architectural rods should serve for the construction of columns. Columns were a central topic in civil architecture that had its origin in Vitruvius’ *Architectura* [15]. He distinguished four *orders of column*, which meant “Types of column”: The *Tuscan*, the *Doric*, the *Ionic*, and the *Corinthian* orders. During the Renaissance Vignola added the *Roman* order in 1562 [14]. Since that time, one speaks about “the five classical orders of column.” Vitruvius already used many special terms to describe typical components of a column, e.g. base (*spira*), shaft (*scapus*), and capital (*capitulum*). In the following, I refer to Goldmann’s system. Thereby I restrict myself to the “bodies” that are relevant for a comparison of the proportions.

Let us look at a column (Fig. 3). Goldmann considered three bodies:
Pedestal [1;2]– *Column* [2; 3]– *Entablature* [3;4].

Each of the bodies again has three parts.

Pedestal: *Base* – *Die* – *Cap*;

Column: *Base* – *Shaft* – *Capital*;

Entablature: *Architrave* – *Frieze* – *Cornice*.

[5; 6] indicates the largest lower radial range;

[7; 8] indicates the largest upper radial range.

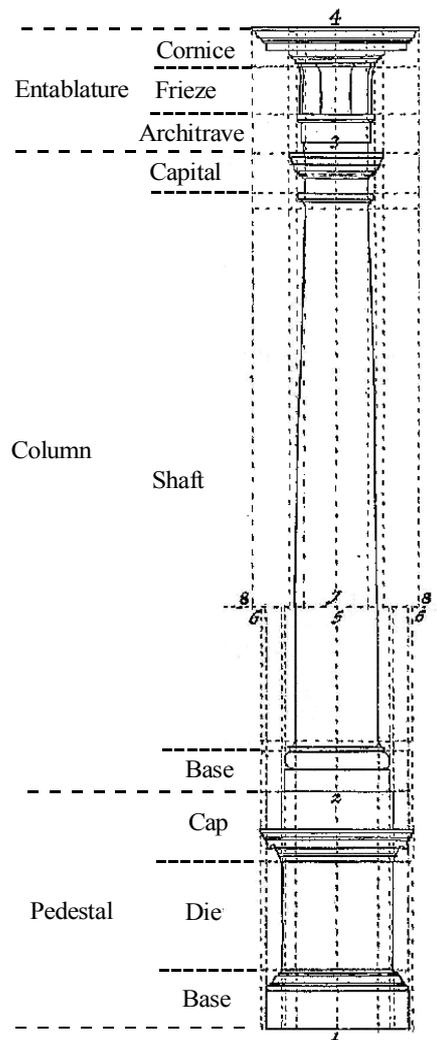


FIGURE 3.
The structure of a column.

In Figure 4, Goldmann presents the five orders of column in comparison using the same diameter for all columns.

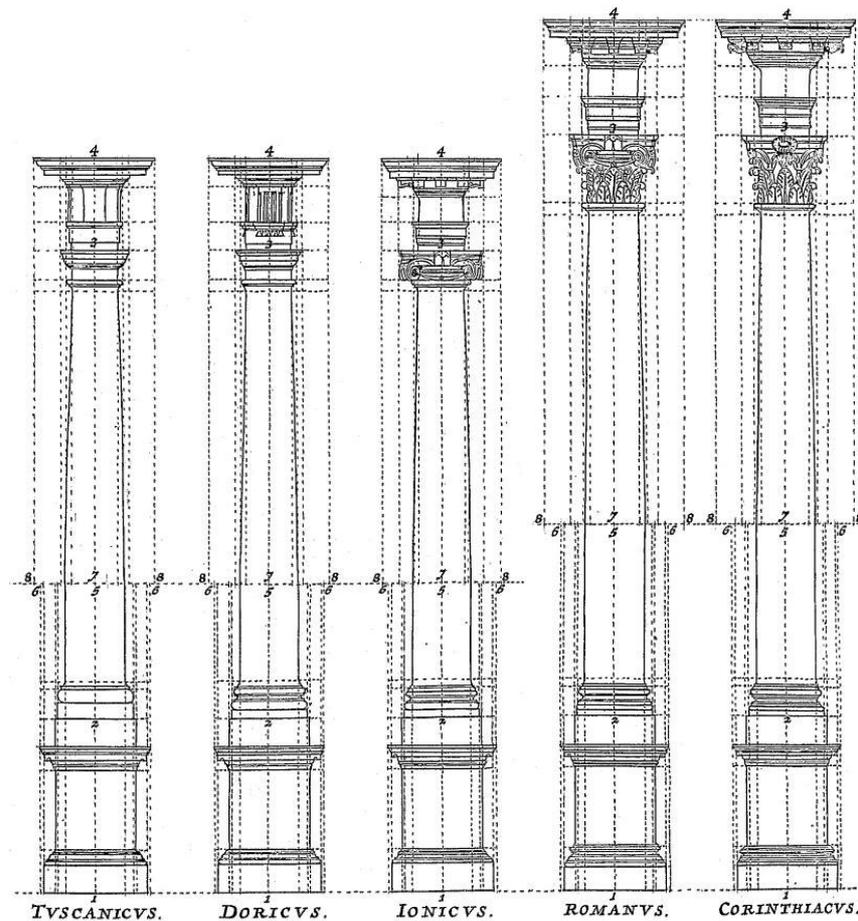


FIGURE 4.

The five classical orders of column ([4], Fig. 8).

For the construction of a column, one needs the heights and the widths of the parts. All these measures are expressed in *modules*. Goldmann takes the lower radius of the column as a measure for columns. Hence, the module is a relative measure. Table 1 shows some measures of the five orders that Goldmann used in his *Tractatus*.

	<i>Tuscan</i>	<i>Doric</i>	<i>Ionic</i>	<i>Roman</i>	<i>Corinthian</i>
Height of pedestal	6	6	6	6	6
Height of column	16	16	16	20	20
Height of entablature	$3 \frac{1}{5}$	$3 \frac{1}{5}$	$3 \frac{1}{5}$	4	4
largest lower radial range	$1 \frac{7}{8}$	$1 \frac{7}{8}$	$1 \frac{7}{8}$	$1 \frac{7}{8}$	$1 \frac{7}{8}$
largest upper radial range	$2 \frac{2}{25}$	$2 \frac{2}{25}$	$2 \frac{2}{15}$	$2 \frac{13}{30}$	$2 \frac{13}{30}$

Table 1.

Measures of the orders in modules ([4], p. 8).

The table shows the measures of the bodies that make the rough structure of a column. Vitruvius and Vignola offer partly different measures, but they do not differ significantly. Therefore, Goldmann may have had in mind his architectural rods with their limited precision. However, the orders more differ in detail as can be seen in Figure 4.

About the construction of columns

The construction of a column of a certain order could be done in different ways. The easiest way was to start with a given length of the module. Then one could *calculate* the different measures of the bodies corresponding to the data in Table 1. In those days, multiplications could be done with the aid of a *proportional compass* [11]. Goldmann had described such an instrument that he called *Proportionatorium* resp. *Ebenpaßer*. He also mentioned it in the title of his *Tractatus*:

„Gebrauch Dehr Baustäbe, Durch dehrer hülffe Die fünf Ordnungen der Bau Kunst aufs aller leichteste, ja behänder und genauer als mit einigem Ebenpaßer, in großer und kleiner Form abgebildet werden.“ [4]

The rods were meant as instruments that would allow mere *constructions* without calculations. In addition, that method was expected to be faster and easier. Goldmann's set of 6 rods contains one *universal rod* for the construction of the rough structure for each of the five orders. The other five *special rods* are provided for the construction of each order's fine structure. The shafts of the rods should have had the length of “*dreyviertheilen eines Rheinländischen Fußes*“. The Rheinian foot measured 31.385 cm ([8], p. 111); three quarter of it is 23.539 cm. (My measure of 23.4 cm differs less than 1%.)

Constructions with the universal rod

Now I want to show how to construct a column with the universal rod (Fig. 5). T., D., I., R., and C. indicate the scales for the measures of the bodies of the five orders. V. designates the scale for the measures of the volutes.

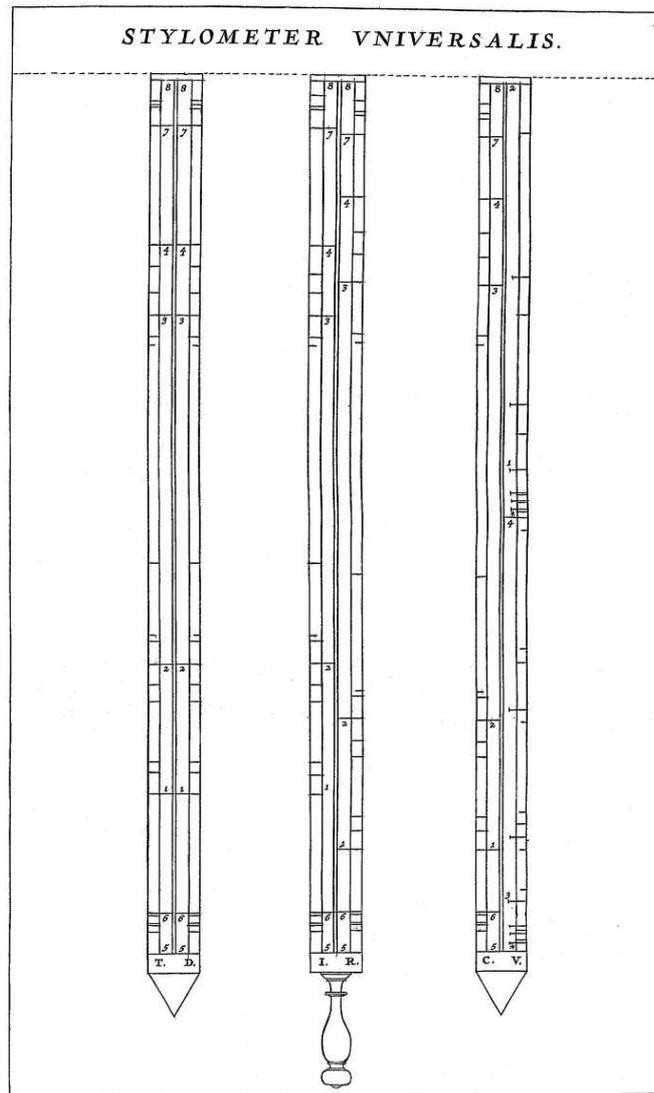


FIGURE 5.
The universal rod ([4], Fig.1).

There are two scales with marks for each order. Some of them are numbered. The following intervals determine for each order:

- [1; 2]: Height of pedestal,
- [2; 3]: height of the column,
- [3; 4]: height of the entablature,
- [5; 6]: largest lower radial range,
- [7; 8]: largest upper radial range.

The intervals correspond to the measures from table 1. The marks between the numbers refer to parts of the bodies. As an example, I show the construction of a Tuscan pedestal in case that the height of the pedestal is smaller than the corresponding line on the rod (reduction).

Given the height h_p of the pedestal. I first draw the vertical line AB with $[AB] = h_p$ (Fig. 6). Then I draw a horizontal line through A. Now I put the universal rod with its Tuscan-scale such that 2 on the scale meets B and 1 lies just on the horizontal line through A. There are 4 unnumbered marks on the scale between 1 and 2. When I draw horizontal lines through these marks, I get the different heights on AB.:

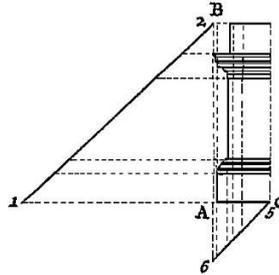


FIGURE 6.

Construction of the Tuscan pedestal (reduction), ([4], Fig. 7).

To find the module I first place the rod with the Tuscan scale in a way, that 5 lies on the horizontal line through A and 6 lies on the line AB. This leads to C. $[AC]$ is the module of the column. Parallel lines to AB through the marks between 5 and 6 lead to different radial ranges. The construction results from the properties of parallel lines. The other two parts are constructed in a similar way.

For the case of an enlargement, Figure 7 shows the use of the architectural rod. One starts with the lines AB, AC, and CD. Then point E is chosen such that $BE = DE$. Now the rod is placed parallel to BD with mark 1 in ED, and mark 2 in EB, etc.

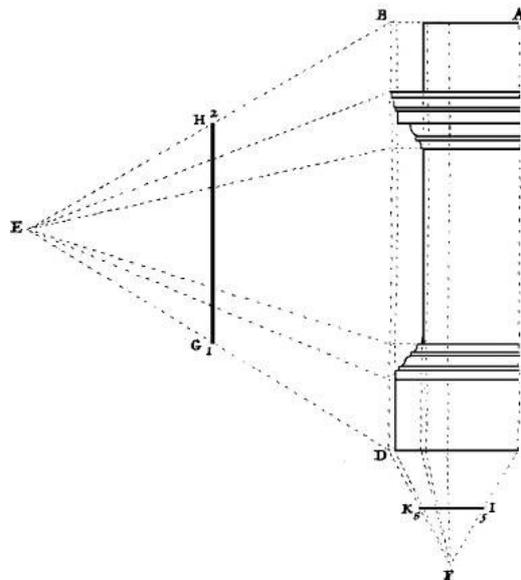


FIGURE 7.

Construction of the Tuscan pedestal (enlargement), ([4], Fig. 7).

The special rods

As an example of the special rods, I will now study the Tuscan rod. Figure 8 shows the three sides of the rod.

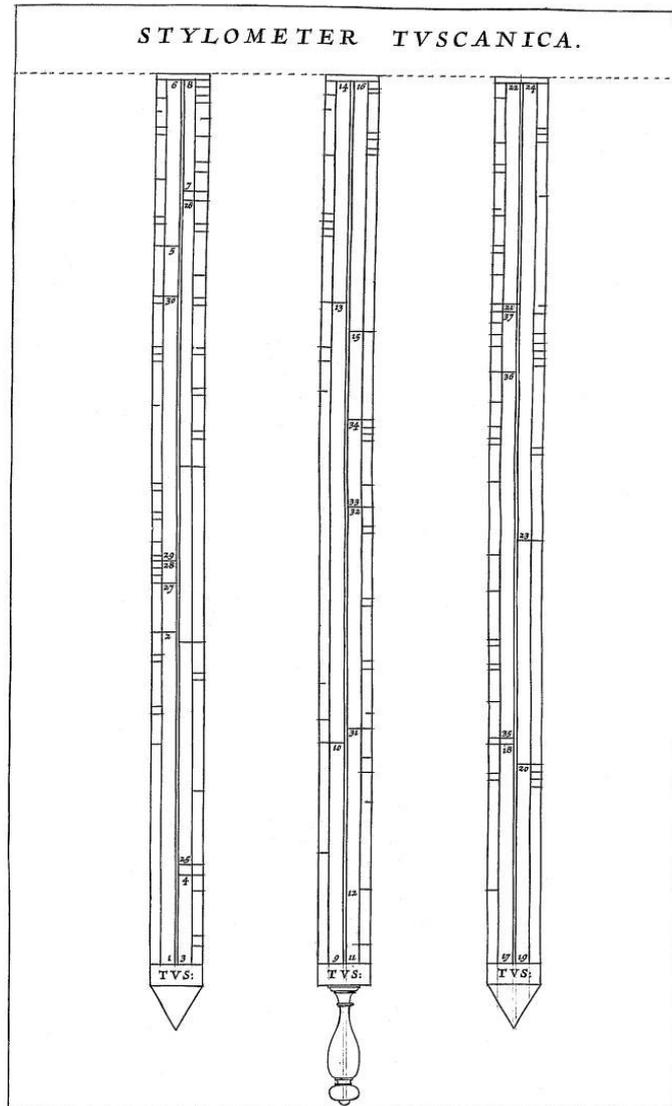


FIGURE 8.
The Tuscan rod (Goldmann 1662, Fig. 2).

The numbers by the sides refer to endpoints of lines with the following meaning:

- [1; 2]: Height of the base of the pedestal.
- [3; 4]: Increase of the base of the pedestal.
- [5; 6]: Height of the cap of the pedestal.
- [7; 8]: Increase of the base of the pedestal.
- [9; 10]: Height of the base of the column.
- [11; 12]: Increase of the base of the column.
- [13; 14]: Height of the capital.

- [15; 16]: Largest range of the capital.
- [17; 18]: Height of the architrave.
- [19; 20]: Largest range of the architrave.
- [21; 22]: Height of the cornice.
- [23; 24]: Largest range of the cornice.

The numbers from 25 to 37 refer to the parapet. One should pay attention that equal numbered intervals on the universal rod and the special rods have a different meaning.

Goldmann’s construction of a volute

The V. on the universal rod refers to the *volute* of the Ionic order (Fig.9).

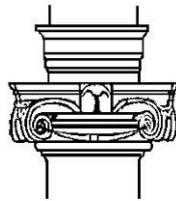


FIGURE 9.
Volute of the Ionic order ([4], Fig. 8).

In his famous construction, Goldmann composes quarters of circles that finally pass over into an inner circle, called the *eye*.

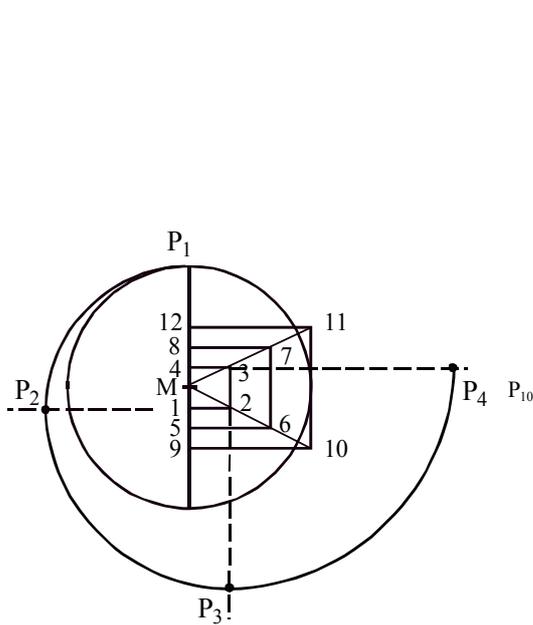


FIGURE 10.
Construction of the volute

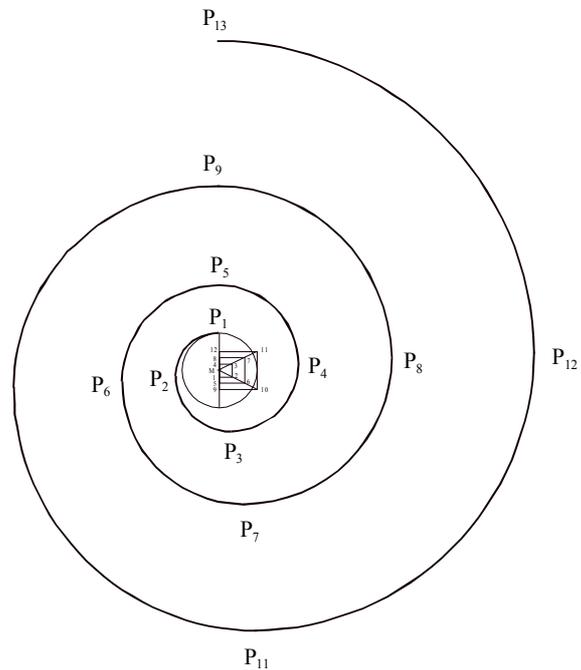


FIGURE 11.
Volute

First, I start the construction without the use of an architectural rod. I start with the eye, a circle with centre M and radius r. The highest point of the volute will then be 9r away from

M. The volute will be composed of 12 quarters of circles. To find their centres I draw three nested squares in the way of Figure 10. Their corners will be the centres. The side of the inner square is $\frac{1}{3}r$, its corners get the numbers 1, 2, 3, 4. The next square has the side $\frac{2}{3}r$, and the corners 5, 6, 7, 8. The third square has the side r , and the corners 9, 10, 11, 12. Now I can draw the quarter-circles. The first one with centre 1 has the radius $\frac{7}{6}r$. It starts in P_1 and ends in P_2 . The second quarter-circle has the distance from 2 to P_2 as radius $r_2 = \frac{9}{6}r$. Continuing this procedure I end up at P_{13} by the 12th quarter-circle with centre 12 and the radius $r_{12} = \frac{54}{6}r = 9r$.

This construction is rather simple. I started with the eye, whereas Goldmann draws from outside to inside because I always got better results by this. In practice, one needs more types of volute that he also dealt with [4].

The universal rod can also help to draw a volute. At the scale V. one finds the intervals [M4], [M8], [M12], [MP₁], [MP₅], [MP₉] and [MP₁₃]. They can be expanded or reduced. However, I doubt that this is an easier method.

In October 2010 the Kulturhistorisches Museum Stralsund exhibited a set of seven architectural rods in an exhibition on astronomical and nautical instruments. This set belongs to the collection of AXEL GRAF VON LÖWEN (1686–1772). It was identified by Dr. Jürgen Hamel as curator of this exhibition.



FIGURE 12
Case of the seven architectural rods from Stralsund
Kulturhistorisches Museum Stralsund (Inv. Nr. A- 2009.206).

A comparison with the rods from München reveals the following differences:

- The rods from Stralsund are shorter than the rods from München: they measure about $\frac{2}{3}$ of them.
- The set from Stralsund has an additional seventh rod. This rod is restricted to the measures of five volutes. The left scale contains the vertical measures (Alt.), and the right scale contains the horizontal measures (Fig. 13).

- Instead of scale V on the universal rod from München, the universal rod from Stralsund has a scale E that contains the intervals [5; 6] and [7; 8] of the five orders. They belong to the radial ranges (gr.: *ekphora*) that may explain the choice of the letter E.
- The special rods from Stralsund have scales that differ in detail from the corresponding scales of the rods from München. Therefore the resulting columns will also differ in detail.

Concerning these properties the rods from Stralsund differ from the rods in Goldmann's *Tractatus*. But nevertheless they can also be called "Goldmann's architectural rods".

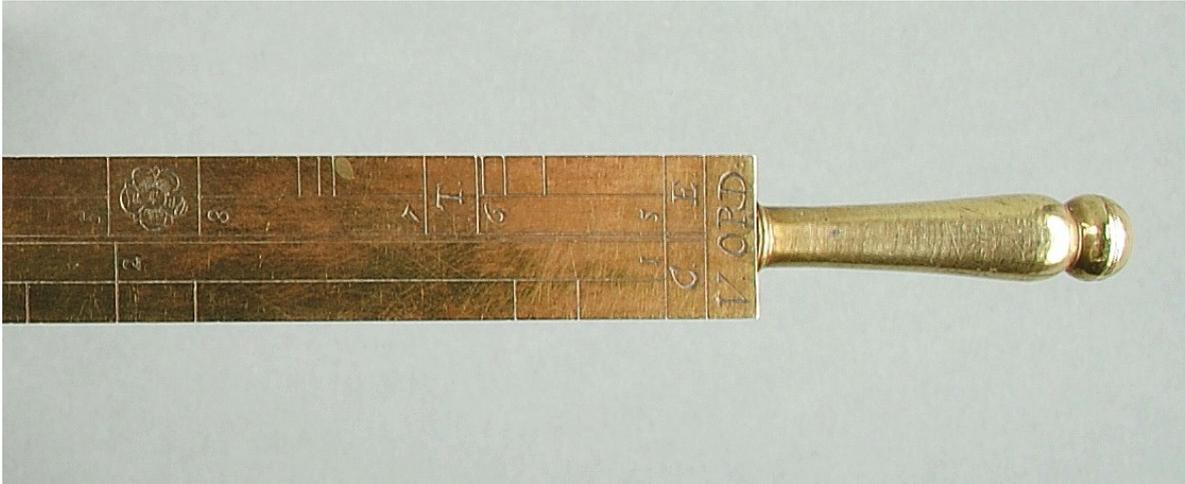


FIGURE 13.
Part of the universal rod from Stralsund,
Kulturhistorisches Museum Stralsund (Inv. Nr. A- 2009.206).

An evaluation of Goldmann's rods

The architectural rods were invented by Nikolaus Goldmann to facilitate the construction of columns. They help to find the different sizes for a given module just by construction, avoiding calculations. However, to find the numbers of the respective intervals on the rod, one needs to look at a table that Goldmann offers in his *Tractatus*. This is a similar procedure as looking for the respective numbers in a table for calculation. Therefore, it is just the procedure of constructing or calculating that differs. In my view, the necessary calculations are rather simple, but precise. The constructions are rather complicated and not very precise. So I can agree to MAYA HAMBLBY: "These rods must be included amongst the interesting examples of specialist drawing instruments which were never adopted into common use." ([7], p. 142).

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