

Slonimsky's Multiplying Device, an impressive Example for Applied Mathematics
Appendix: Slonimsky's theorem explained

Stephan Weiss

In the minutes of the Public Meeting of Royal Academy of Sciences, St. Petersburg, on April 10, 1845, Slonimsky's theorem is explained thus (free translation by me):

"The theorem found by Mr. Slonimsky deals with a highly strange property of simple numbers: assumed an integer number with as many figures as you like is multiplied by 2, 3 up to 9 and the results are written down one below the other without moving the results from place to place, then we get whole vertical columns with nine figures. Shortly we name the sequence of numbers in such a column a form. Due to Slonimsky's theorem the amount p of different forms is expressed with the formula $p = 10(q + 1)$ with q as the amount of all proper and different fractions with denominators 2, 3 to 9. In that case a simple calculation shows that there will never be more than 280 columns which are different in their form. This relatively small number led Slonimsky to the idea to build a calculating device and the success complied his hypothesis thoroughly."

$$1. \left\{ \begin{array}{l} \frac{1}{9} \quad \frac{2}{9} \quad \frac{4}{9} \quad \frac{5}{9} \quad \frac{7}{9} \quad \frac{8}{9} \\ \frac{1}{8} \quad \frac{3}{8} \quad \frac{5}{8} \quad \frac{7}{8} \\ \frac{1}{7} \quad \frac{2}{7} \quad \frac{3}{7} \quad \frac{4}{7} \quad \frac{5}{7} \quad \frac{6}{7} \\ \frac{1}{6} \quad \frac{5}{6} \\ \frac{1}{5} \quad \frac{2}{5} \quad \frac{3}{5} \quad \frac{4}{5} \\ \frac{1}{4} \quad \frac{3}{4} \\ \frac{1}{3} \quad \frac{2}{3} \\ \frac{1}{2} \end{array} \right.$$

FIGURE 11. The twenty seven fractions used by Crelle

The mathematician August Leopold Crelle proved Slonimsky's theorem one year later [5]. For his explanations he uses the 27 fractions (FN5) shown in fig. 11 or, ordered by value: $1/9, 1/8, 1/7, 1/6, 1/5, 2/9, 1/4, 2/7, 1/3, 3/8, 2/5, 3/7, 4/9, 1/2, 5/9, 4/7, 3/5, 5/8, 2/3, 5/7, 3/4, 7/9, 4/5, 5/6, 6/7, 7/8, 8/9$.

Shortened, Crelle's proof runs as follows: each carry sequence is determined by all figures that are placed right of it within the number Z . So if one writes down any decimal fraction f with $0 < f < 1$, followed from top to bottom by its products $2*f$ to $9*f$, then the integer parts of all multiples on the left side form a carry sequence that belongs to the figure sequence in the first line. In fig. 12 the complete table is listed for the decimal fraction $f = 0.274$, which gives $(0,0,0,1,1,1,1,2,2)$ as carry sequence. This one equals s_8 in fig. 5.

0.	2	7	4	
0.	5	4	8	*2
0.	8	2	2	*3
1.	0	9	6	*4
1.	3	7	0	*5
1.	6	4	4	*6
1.	9	1	8	*7
2.	1	9	2	*8
2.	4	6	6	*9

FIGURE 12. Table to isolate a carry sequence

Next Crelle shows that all decimal fractions $0 < f < 1$ can be replaced by 28 intervals between the 27 proper fractions given before (see fig. 5 right half) and that every interval stands for a carry sequence that doesn't change. The integer parts of the multiples of these fractions give the carry sequences looked for. The replacement of decimal fractions with proper fractions eases to estimate the integer part of the result when multiplying with 2, 3,...

The example for s_2 :

for $1/9 \leq x < 1/8$ we get $\text{int}(2 \cdot 1/9) = 0$ up to $\text{int}(8 \cdot 1/9) = 0$; $9 \cdot 1/9 = 1$;

The example for s_{28} :

for $8/9 \leq x < 1$ we get $\text{int}(2 \cdot 8/9) = 1$; $\text{int}(3 \cdot 8/9) = 2$; $\text{int}(4 \cdot 8/9) = 3$ and so on.

The multiplier 10 in the formula above equals the amount of figures 0 to 9, because to all partial products of each figure 0 to 9 all 28 completion sequences can be added, which gives not more than 280 different columns in the multiplication table.

Sources

Fig. 11: Crelle [5], for bibliographic data see main article,

Fig. 12: by the author.

Footnotes

6. Also called Farey sequence $F(n=9)$ without the members $0/1$ and $1/1$. "The Farey sequence of order n is the sequence of completely reduced fractions between 0 and 1 which, when in lowest terms, have denominators less than or equal to n , arranged in order of increasing size" (from Wikipedia.org).