



Basic Slide Rule Operation for Scientists and Engineers

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Scientists and Engineers often must solve problems containing very large and very small numbers; in my case, analog electronic circuit design, frequency ranges from 0 (dc) to in excess of 10 gigahertz (10^9 Hz), current from amperes to nanoamperes (10^{-9}), capacitors from farads to picofarads (10^{-12}), inductors from henries to nanohenries (10^{-9}) and resistors from ohms to megohms (10^6). We usually expressed large and small numbers as; $N.nnnE^{(+/-)x}$, where $E^{(+/-)x} = 10^{(+/-)x}$.

The Oughtred Society's Slide Rule Reference Manual, besides showing a large selection of slide rules (some common and many rare), has an excellent introduction to basic slide rule operation. So why this article? I taught evening classes in solid state circuit design at our local community college from 1967 to 2000, and during the pre calculator days, around 1974, I assumed my students had a firm grasp on slide rule operation. As a slide rule review I handed out a basic overview, much like you see here. This presentation stresses slide rule solutions with large and small numbers, $N.nnnE^{(+/-)x}$, and in many instances takes a different approach to that presented in the Oughtred Manual; this article should be considered as an expansion of the Oughtred Manual and not competition!

Most of my presentation is straightforward and easily committed to memory; who knows when you will be called on to solve a problem during a power outage and find your calculator batteries have died. Note the word "Most" at the beginning of this paragraph. The exceptions are the Log-Log scales ($y = a^N$, $N = \text{Log}_a y$, and $a = \sqrt[N]{y}$); the solution process is simple: however, which Log-Log scale to use can be problematic. The solution to this problem and the use of the other primary slide rule scales, are presented in the following pages.

I

Basic Multiplication, Division, and Decimal Point Determination

I Multiplication; $(a)(b) = c$

- Place the C scale index over a on the D scale; under b on the C scale read $(a)(b) = c$ on the D scale.
- Place b on the CI scale (inverted C scale; $1/b$ on C = b on CI); under the CI index read $(a)(b) = c$ on the D scale.

II Division, $a/b = c$

- Place b on the C scale over a on the D scale; under C index read $a/b = c$ on the D scale.
- Place the CI index over a on the D scale; Under b the CI index read $a/b = c$ on the D scale.

III Determine the decimal point;

Write all numbers as $N.nnnE^{(+/-)x}$ ($E^{(+/-)x} = 10^{(+/-)x}$) and approximate the solution.

$$(a)(b)/c = d;$$

$$d = (30100)(0.0862)/6020 = (3.01E^{+4})(8.62E^{-2})/6.02E^{+3} \approx (3E^{+4})(10E^{-2})/5E^{+3}$$

$$d \approx 6E^{(+4-2-3)} = 6E^{-1} = 0.6$$

Using the C/D sales;

$$(3.01E^{+4})(8.62E^{-2})/6.02E^{+3} \approx 4.32E^{-1} = 0.432 \quad (0.431)$$

The DF/CF Scales

D/C scales folded at π , some older slide rules folded at $\sqrt{10}$

You can multiply and divide on the DF/CF scales just as we do on the D/C scales.

- $N.nnnE^{(+/-)x}$ on the D scale = $(\pi)N.nnnE^{(+/-)x}$ on the DF scale.
- $N.nnnE^{(+/-)x}$ on the DF scale = $N.nnnE^{(+/-)x}/\pi$ on the D scale.

Gauge Marks

Slide rules usually have gauge marks representing various constants; π for example. See the Oughtred Society Slide Rule Reference Manual for a listing of the most common and the Oughtred Society's Pocketbook of the Gauge Marks by Panagiotis Venetsianos for a complete listing.

II

Basic Trigonometry

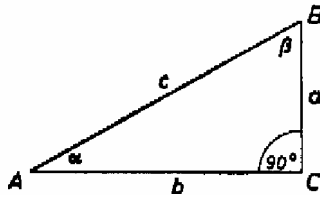


Figure 1.
Triangle with angles and sides labeled

$\text{Sin}\alpha = a/c$; α on the S scale = $\text{Sin}\alpha$ on the C scale (C scale limits 0.1 to 1).

$\alpha = \text{Sin}^{-1}(a/c)$; a/c on the C scale = α on the S scale.

For small angles

α on the ST/SRT scale = $\text{Sin}\alpha$ on the C scale (C scale limits 0.01 to 0.1)

$\text{Cos}\alpha = b/c$; α on the red (Cos) S scale = $\text{Cos}\alpha$ on the C scale (C scale limits 0.1 to 1).

$\alpha = \text{Cos}^{-1}(b/c)$; b/c on the C scale = α on the red(Cos) S scale.

$\text{Tan}\alpha = a/b < 1$; $\alpha < 45^\circ$, α on the black T scale = $\text{Tan}\alpha$ on the C scale (C scale limits 0.1 to 1)
 $a/b > 1$ $\alpha > 45^\circ$, α on the red T scale = $\text{Tan}\alpha$ on the CI scale (CI scale limits 1 to 10)

$\alpha = \text{Tan}^{-1}(a/b)$; $a/b < 1$ on the C scale = $\text{Tan}\alpha$ ($\alpha < 45^\circ$) on the black T scale.

$a/b > 1$ on the CI scale = $\text{Tan}\alpha$ ($\alpha > 45^\circ$) on the red T scale.

For small angles

$\text{Sin}\alpha$ (in degrees and radians) = $\text{Tan}\alpha$ (in degrees and radians) on the ST/SRT scale
(scale limits 0.01 to 0.1)

III

A Scale

$$x^2; \sqrt{x}$$

Two Decade, 1 to 10 to 100

Note; we can multiply and divide using A/B scales with reduced accuracy

$$D^2$$

$$\text{Note } 10^{(+/-)x} = E^{(+/-)x}$$

D.dddE^{(+/-)x} on the D scale = [(D.ddd)² on the A scale]E^{(+/-)2x}; Note the 2x.

$$D = 2.682E^{-8}; D^2 \approx 7.2E^{-2(8)} = 7.2E^{-16} \quad (7.193E^{-16})$$

$$D = 4.385E^4; D^2 \approx 19.2E^{2(4)} = 19.2E^8 \quad (19.23E^8)$$

$$\sqrt{A}$$

A on the A scale = \sqrt{A} on the D scale

We must first determine which decade A scale to use; left (1 to 10), or right (10 to 100). Write A as A.aaaE^{(+/-)x}. If x is even (0, 2, 4, 6, etc.) use the left A scale, if x is not even, shift the decimal point one place to the right, A = AA.aa E^{(+/-)x-1} and use the right A scale;

a) x is even, use left A scale; \sqrt{A} on the A scale = [(\sqrt{A} .aaa) on the D scale]E^{(+/-)x/2}

Note the x/2

b) x is not even, shift decimal one place to the right, A = AA.aa E^{(+/-)x-1}

\sqrt{A} on the A scale = [(\sqrt{AA} .aa) on the D scale] E^{[(+/-)x-1]/2} and use the right A scale.

Note the (x-1)/2

We can easily tell if a shift in decimal point is necessary.

- 1) A = 3.725E⁺⁵. x = +5, is not even so shift the decimal one place to the right, A = 37.25E⁺⁵⁻¹ = 37.25E⁺⁴ (right A scale). $\sqrt{A} = [(\sqrt{37.25}) \text{ on the D scale}]E^{+4/2} \approx 6.1E^2 \quad (6.103E^2)$.
- 2) A = 6.817E⁻¹³. x = -13 is not even so again we must shift the decimal point one place to the right; A = 68.17E⁻¹³⁻¹ = 68.17E⁻¹⁴ (right A scale); $\sqrt{A} = [(\sqrt{68.17}) \text{ on the D scale}]E^{-14/2} \approx 8.26E^{-7} \quad (8.256E^{-7})$.

We will now write A by inspection

- 3) A = 5.542E⁺⁶ (left A scale); $\sqrt{A} \approx 2.36E^3 \quad (2.354E^3)$.
- 4) A = 8.249E⁻¹⁷ = 82.49E⁻¹⁸ (right A scale); $\sqrt{A} \approx 9.07E^{-9} \quad (9.082E^{-9})$.
- 5) A = 2.262E⁻⁵ = 22.62E⁻⁶ (right A scale); $\sqrt{A} \approx 4.75E^{-3} \quad (4.756E^{-3})$.
- 6) A = 4.545E⁻¹² (left A scale); $\sqrt{A} \approx 2.13E^{-6} \quad (2.132E^{-6})$.
- 7) A = 3.258E⁺¹¹ = 32.58E⁺¹⁰ (right A scale); $\sqrt{A} \approx 5.71E^{+5} \quad (5.708E^{+5})$.

IV

K Scale

$$x^3; \sqrt[3]{x}$$

Three Decades; 1 to 10 to 100 to 1,000

$$D^3$$

$$\text{Note } 10^{(+/-)x} = E^{(+/-)x}$$

D.dddE^{(+/-)x} on the D scale = [(D.ddd)³ on the K scale]E^{(+/-)3x}; Note the 3x.

$$D = 4.879E^{+4}; D^3 \approx 116E^{+3(4)} = 116E^{+12} \quad (116.14E^{+12})$$

$$D = 2.385E^{-3}; D^3 \approx 13.6E^{-3(3)} = 13.6E^{-9} \quad (13.566E^{-9})$$

$$D = 1.738E^{-4}; D^3 \approx 5.23E^{-3(4)} = 5.23E^{-12} \quad (5.2499E^{-12})$$

$$\sqrt[3]{K}$$

K on the proper K scale = $\sqrt[3]{K}$ on the D scale

We must determine which K decade scale to use. K must be K.kkkE^{(+/-)x} for the left K scale, KK.kkE^{(+/-)x} for the center K scale, or KKK.kE^{(+/-)x} for the right K scale, where x must be a multiple of 3; 0, 3, 6, 9, etc.

$$\text{Write } K = K.kkkE^{(+/-)x}$$

- 1) If x is a multiple of 3, no problem; use the left K scale

$$\sqrt[3]{K} = [(\sqrt[3]{K.kkk}) \text{ on the D scale}]E^{[(+/-)x]/3}; \text{ Note the } x/3.$$

- 2) If x isn't a multiple of 3, move the decimal point one place to the right;

$$K = KK.kkE^{(+/-)x-1}$$

if (+/-x) - 1 is a multiple of 3, again no problem; use the center K scale.

$$\sqrt[3]{K} = [(\sqrt[3]{KK.kk}) \text{ on the D scale}]E^{[(+/-)x-1]/3}; \text{ Note the } [(x)-1]/3$$

- 3) If [(+/-x)-2] still is not a multiple of 3, move the decimal point one more time to the right; and use the right K scale; K = KKK.kE^{(+/-)x-2}

$$\sqrt[3]{K} = [(\sqrt[3]{Kkk.k}) \text{ on the D scale}]E^{[(+/-)x-2]/3}$$

Note the [(x-2)/3]

In practice it is easy to find $\sqrt[3]{K}$ as shown below;

- 1) K = 4.658E⁻⁵. The exponent, -5, isn't a multiple of 3 so shift the decimal one place to the right; K = 46.58E⁻⁶. The exponent is a multiple of 3, thus

$$\sqrt[3]{K} = [(\sqrt[3]{46.58}) \text{ on the D scale}]E^{-6/3} \approx 3.6E^{-2} \quad (3.598E^{-2})$$

- 2) K = 6.592E⁻⁷. The exponent, -7, is not a multiple of 3 so shift the decimal one place to the right, K = 65.92E⁻⁸. The exponent still is not a multiple of 3, so shift the decimal to the right one more time; K = 659.2E⁻⁹ and the exponent is a multiple of 3, use the right K scale.

$$\sqrt[3]{K} = [(\sqrt[3]{659.2}) \text{ on the D scale}]E^{-9/3} \approx 8.7E^{-3} \quad (8.703E^{-3})$$

- 3) K = 7.385E⁺⁵ = 738.5E⁺³ (right K scale);

$$\sqrt[3]{K} = [(\sqrt[3]{738.5}) \text{ on the D scale}]E^{+3/3} \approx 90.5 \quad (90.39)$$

- 4) K = 8.752E⁺¹³ = 87.52E⁺¹² (center K scale);

$$\sqrt[3]{K} = [(\sqrt[3]{87.52}) \text{ on the D scale}]E^{+12/3} \approx 4.44E^{+4} \quad (4.4399E^{+4})$$

V

The Log L scale

$$N = \text{Log}_{10}y$$

$$N = \text{Log}_{10}y$$

Note; $E^x = 10^x$

Write y as; $y = Y.yyyE^C$ where $Y.yyy > 0$

$$N = \text{Log}_{10}y = \text{Log}_{10}(Y.yyyE^C) = \text{Log}_{10}(E^C) + \text{Log}_{10}(Y.yyy) = C \text{ (characteristic)} + (\text{Log}_{10}(Y.yyy) = 0.mmm \text{ mantissa})$$

- 1) place cursor over $y = Y.yyy$ on the D scale
- 2) read the mantissa ($\text{Log}_{10}(Y.yyy) = 0.mmm$) below cursor on the L scale
- 3) $N = \text{characteristic (C)} + \text{mantissa (Log}_{10}(Y.yyy) = 0.mmm)$

$y > 1; N > 0$ (positive)

Let $y = 2,500 = 2.5E^3$ (characteristic $C = 3$)

$$N = \text{Log}_{10}y = \text{Log}_{10}(Y.yyyE^C) = \text{Log}_{10}(2.5E^3) = \text{Log}_{10}(E^3) + \text{Log}_{10}(2.5) = 3 \text{ (characteristic)} + 0.39794 \text{ (mantissa)} = 3.39794$$

- 1) place cursor over $y = Y.yyy = 2.5$ on the D scale
- 2) read the mantissa, ($\text{Log}_{10}(Y.yyy) = \text{Log}_{10}(2.5) = 0.mmm \approx 0.398$) below cursor on the L scale
- 3) $N = \text{characteristic (C} = 3) + \text{mantissa (Log}_{10}(Y.yyy) = 0.mmm \approx 0.398) \approx 3 + 0.398 = 3.398 \text{ (3.39794)}$

$0 < y < 1; N < 0$ (negative)

Let $y = 0.0025 = 2.5E^{-3}$ (characteristic $C = -3$)

$$N = \text{Log}_{10}y = \text{Log}_{10}(2.5E^{-3}) = \text{Log}_{10}(E^{-3}) + \text{Log}_{10}(2.5) = -3 \text{ (characteristic)} + 0.39794 \text{ (mantissa)} = -2.60206$$

- 1) place cursor over $y = Y.yyy = 2.5$ on the D scale
- 2) read the mantissa, ($\text{Log}_{10}(Y.yyy) = \text{Log}_{10}(2.5) = 0.mmm \approx 0.398$) below cursor on the L scale
- 3) $N = \text{characteristic (C} = -3) + \text{mantissa (Log}_{10}(Y.yyy) = 0.mmm \approx 0.398) \approx -3 + 0.398 \approx -2.602 \text{ (-2.60206)}$

Note; Log Tables give N as $[(10 + C) + \text{mantissa (0.mmm)}] - 10$. For our example, $C = -3$
 $N = [(10 - 3) + 0.39794 \text{ (mantissa)}] - 10 = [7 + 0.39794] - 10 = 7.39794 - 10 = (-2.60206)$

$$N = \text{Log}_a y$$

$$N = \text{Log}_a y = (\text{Log}_{10}y) / (\text{Log}_{10}a)$$

This is the method for finding N using a calculator

VI

The L Scale

$$y = 10^N$$

$$y = 10^N$$

Note; $E^x = 10^x$

Write N as; C.mmm, where C is the characteristic and 0.mmm is the mantissa

$$y = 10^N = [10^{C(\text{characteristic})}] [10^{0.\text{mmm}(\text{mantissa})}] = [10^{0.\text{mmm}}] [10^C] = Y.\text{yyy}E^C$$

- 1) place cursor over the mantissa (0.mmm) on L scale
- 2) read ($10^{0.\text{mmm}(\text{mantissa})} = Y.\text{yyy}$) below the cursor on the D scale
- 3) $y = 10^N = 10^{0.\text{mmm}(\text{mantissa})} [10^{C(\text{characteristic})}] = 10^{0.\text{mmm}} [10^C] = Y.\text{yyy}E^C$

y>1; N>0 (positive)

$$\text{Let } N = 3.39794 = 3(\text{characteristic, } C) + 0.39794(\text{mantissa, } 0.\text{mmm})$$

- 1) place cursor over the mantissa (0.mmm) = 0.39794 on L scale
- 2) read ($10^{0.\text{mmm}(\text{mantissa})} = Y.\text{yyy}$) ≈ 2.5 below the cursor on the D scale
- 3) $y = 10^N = 10^{0.\text{mmm}(\text{mantissa})} [10^C(\text{characteristic})] = 10^{(0.39794)} [10^3] \approx 2.5E^3 (2.5000E^3)$

0<y<1; N<0 (negative)

Write N as; $N = -N.\text{nnn} = -N - 0.\text{nnn}$ and we must convert $-N - 0.\text{nnn}$ to;
characteristic (C) + mantissa (0.mmm) as shown below.

$$\text{characteristic (C)} = N - 1$$

$$\text{mantissa (0.mmm)} = 1 - 0.\text{nnn}$$

$$\text{Thus } y = 10^N = 10^{-N.\text{nnn}} = [10^{\text{characteristic } C}] [10^{\text{mantissa (0.mmm)}}] = [10^{(N-1)}] [10^{(1-0.\text{nnn})}]$$

$$\text{Or; } y = [10^{0.\text{mmm}}] [10^C]$$

$$\text{Let } N = -2.60206 = -N - 0.\text{nnn} = -2 - 0.60206$$

$$\text{characteristic (C)} = N - 1 = -2 - 1 = -3$$

$$\text{mantissa (0.mmm)} = 1 - 0.\text{nnn} = 1 - 0.60206 = 0.39794$$

$$y = 10^{0.39794} E^{-3} = 2.500E^{-3}$$

- 1) place cursor over the mantissa (0.mmm) = 0.39794 on L scale
- 2) read ($10^{0.\text{mmm}(\text{mantissa})} = Y.\text{yyy}$) ≈ 2.5 below the cursor on the D scale
- 3) $y = 10^N = 10^{0.\text{mmm}(\text{mantissa})} [10^C(\text{characteristic})] = 10^{(0.39794)} [10^{-3}] \approx 2.5E^{-3} (2.5000E^{-3})$

$$y = a^N = 10^{N \log_a}$$

where $N \log_a = N \log_{10} a$

The Log Log scales solve this example quite easily.

VII

The Log-Log Scales Log Log Scale Nomenclature, Scale Limits and Operation $N = \text{Ln} y = \text{Log}_e y; y = e^{\pm N}$

Log Log Scale Nomenclature and Limits (inverted, LL0, scales in red)						
Note; LL/LLO are mated scales; Z on LLZ scale = 1/Z on LL0Z scale; 1/Z on LL0Z scale = Z on LLZ scale						
K&E 4080/81/83	K&E Deci-Lon	Nestler 0292	Post 1461	Chinese 1003	Scale Limits e^N	N limits for the D scale
LL3	Ln3	LL3	LL3	\ln_1	$e^1 \rightarrow e^{10}$	$1 < N < 10$
LL03	Ln-3	LL ₀ 3	LL/3	$\ln_{.1}$	$e^{-1} \leftarrow e^{-10}$	$-1 < N < -10$
LL2	Ln2	LL2	LL2	\ln_0	$e^{0.1} \rightarrow e^1$	$0.1 < N < 1$
LL02	Ln-2	LL ₀ 2	LL/2	\ln_0	$e^{-0.1} \leftarrow e^{-1}$	$-0.1 < N < -1$
LL1	Ln1	LL1	LL1	$\ln_{.1}$	$e^{0.01} \rightarrow e^{0.1}$	$0.01 < N < 0.1$
LL01	Ln-1	LL ₀ 1	LL/1	$\ln_{.1}$	$e^{-0.01} \leftarrow e^{-0.1}$	$-0.01 < N < -0.1$
	Ln0	LL0	LL0	$\ln_{.2}$	$e^{0.001} \rightarrow e^{0.01}$	$0.001 < N < 0.01$
	Ln-0	LL ₀ 0	LL/0	$\ln_{.2}$	$e^{-0.001} \leftarrow e^{-0.01}$	$-0.001 < N < -0.01$
$N = \text{Ln} y$						
$y > 1$ (on LLy scale) ; $N > 0$ positive (on D scale)						
<ol style="list-style-type: none"> 1) place cursor over y on LLy scale 2) under cursor read $+N = \text{Ln} y$ on D scale 3) decimal point same as $+N$ ($N > 0$) scale limits for the LLy scale number Let $y = 1.35$ (LL2 scale); D scale limits $0.1 < N < 1$ $N = \text{Ln}(1.35) \approx 0.3$ (0.30010) 						
$0 < y < 1$ (on LL0y scale); $N < 0$ negative (on D scale)						
<ol style="list-style-type: none"> 1) place cursor over y on LL0y scale 2) under cursor read $-N = \text{Ln} y$ on D scale 3) decimal point same as $-N$ (N scale limits for the LL0y scale number) Let $y = 0.74074$ (LL02 scale); D scale limits $-0.1 < N < -1$ $N = \text{Ln}(0.74074) \approx -0.3$ (-0.30011) 						
$y = e^{\pm N}$						
$y > 1$ (on LLy scale); $N > 0$ positive (on D scale)						
<ol style="list-style-type: none"> 1) place cursor over N on D scale 2) under cursor read $y = e^{+N}$ on LLy scale with $+N$ scale limits Let $N = 0.30011$; D scale limits $0.1 < N < 1$ (LL2 scale) y (LL2) = $e^{0.30011} \approx 1.35$ (1.3499) 						
$0 < y < 1$ (on LL0y scale); $N < 0$ negative (on D scale)						
<ol style="list-style-type: none"> 1) place cursor over N on D scale 2) under cursor read $y = e^{-N}$ on LL0y scale with N scale limits Let $N = -0.30011$; D limits $-0.1 < N < -1$ (LL02 scale) y (LL02) = $e^{-0.30011} \approx 0.7405$ (0.74074) 						

VIII

Log-Log Scale Summary

The LL/LL0 scales are mated; x the LLx scale = 1/x on the LL0x scale, and vice versa

The LL/LL0 scales as a function of the quadrant

<p style="text-align: center;">II</p> <p>$y > 1$; $N < 0$ (negative); $0 < a < 1$</p> $y = a^{-N} = 1/a^N$ $N = \log_a y$ $a = y^{-1/N} = 1/y^{1/N} = 1/({}^N\sqrt{y})$ <p style="text-align: center;">y on LLy scale -N on C scale a on L0a scale</p>	<p style="text-align: center;">I</p> <p>$y > 1$; $N > 0$ (positive); $a > 1$</p> $y = a^N$ $N = \log_a y$ $a = y^{1/N} = {}^N\sqrt{y}$ <p style="text-align: center;">y on LLy scale +N on C scale a on LLa scale</p>
<p style="text-align: center;">III</p> <p>$0 < y < 1$; $N < 0$ (negative); $a > 1$</p> $y = a^{-N} = 1/a^N$ $N = \log_a y$ $a = y^{-1/N} = 1/(y^{1/N})$ <p style="text-align: center;">y on LL0y scale -N on C scale a on LLa scale</p>	<p style="text-align: center;">IV</p> <p>$0 < y < 1$; $N > 0$ (positive); $0 < a < 1$</p> $y = a^N$ $N = \log_a y$ $a = y^{1/N} = {}^N\sqrt{y}$ <p style="text-align: center;">y on LL0y scale +N on C scale a on LL0a scale</p>

IX

Log-Log Scales

Power

$$y = a^{+/-N}; y = a^{(-N)} = 1/a^N$$

The LL and LL0 scales mated; Z on LL = 1/Z on LL0, Z on LL0 = 1/Z on LL

The solution scales depend on which C scale index, left or right, is used

$$y = a^N; y = a^{(-N)} = 1/a^N$$

- 1) move cursor to a on LLa/LL0a scale
- 2) move slide; C index under cursor
- 3) move cursor to N on C scale
- 4) determine the proper LLy/LL0y scale as discussed below
- 5) read $y = a^N$ under cursor on proper LLy/LL0y scale

LL/LL0 Solution Scales

$$a > 1$$

N > 0; a on LLa scale, read y on LLy scale

N < 0; a on LLa scale, read y on LL0y scale

$$0 < a < 1$$

N > 0; a on LL0a scale, read y on LL0y scale

N < 0; a on LL0a scale, read y on LLy scale

LLa/LL0a solution scales

Write N as N.nnnE^{(+/-)x} ($E^{(+/-)x} = 10^{(+/-)x}$)

$$N = 51.3 = 5.13E^1, x = 1; N = 0.00329 = 3.29E^{-3}, x = -3$$

Left C index over a on LLa/LL0a scale
knowing NE^{(+/-)x}

LLy/LL0y solution scale = LLa/LL0a scale + x

Right C index over a on LLa/LL0a scale
knowing NE^{(+/-)x}

LLy/LL0y solution scale = LLa/LL0a scale + x + 1

$$y = a^N$$

a > 1 on LLa scale; N > 0 (positive) on C scale; y > 1 on LLy scale

Let a = 1.02 (LL1 scale) and N = 1.5124E²; x = 2
LLy scale = LLa scale + 1 = 1 + 2 = 3 (LL3 scale)
y (LL3) = (1.02)^(1.5124E2) ≈ 20 (20.000)

Let a = 7 (LL3 scale) and N = 7.1447E⁻³; x = -3
LLy scale = LLa scale + x + 1 = 3 - 3 + 1 = 1 (LL1 scale)
y (LL1) = 7^(7.1447E-3) ≈ 1.014 (1.0140)

$$a = y^{(-N)} = 1/y^{(N)}$$

a > 1 on LLa scale; N < 0 (negative) on C scale; 0 < y < 1 on LL0y scale

Let a = 1.02 (LL1 scale) and N = -1.5124E²; x = 2
LL0y scale = LLa scale + 1 = 1 + 2 = 3 (LL03 scale)
y (LL03) = (1.02)^(-1.5124E2) ≈ 0.05 (0.05000)

Let a = 7 (LL3 scale) and N = -7.1447E⁻³; x = -3
LL0y scale = LLa scale + x + 1 = 3 - 3 + 1 = 1 (LL1 scale)
y (LL1) = 7^(-7.1447E-3) ≈ 0.9862 (0.98619)

$$y = a^N$$

0 < a < 1 on LL0a scale; N > 0 (positive) on C scale; 0 < y < 1 on LL0y scale

Let a = 0.9804 (LL01 scale) and N = 1.5124E²; x = 2
LL0y scale = LL0a scale + 1 = 1 + 2 = 3 (LL03 scale)
y (LL03) = (0.9804)^(1.5124E2) ≈ 0.05 (0.0500)

Let a = 0.14286 (LL03 scale) and N = 7.1447E⁻³; x = -3
LL0y scale = LL0a scale + x + 1 = 3 - 3 + 1 = 1 (LL01 scale)
y (LL01) = (0.14286)^(7.1447E-3) ≈ 0.9862 (0.98619)

$$= y^{-N} = 1/a^N$$

0 < a < 1 on LL0a scale; N < 0 (negative) on C scale; y > 1 on LLy scale

Let a = 0.9804 (LL01 scale) and N = -1.5124E²; x = 2
LLy scale = LL0a scale + 1 = 1 + 2 = 3 (LL3 scale)
y (LL3) = (0.9804)^(-1.5124E2) ≈ 20 (20.000)

Let a = 0.14286 (LL03 scale) and N = -7.1447E⁻³; x = -3
LLy scale = LL0a scale + x + 1 = 3 - 3 + 1 = 1 (LL1 scale)
y (LL1) = (0.14286)^(-7.1447E-3) ≈ 1.014 (1.0140)

X

The Log-Log Scales

Roots

$$a = y^{(1/N)} = \sqrt[N]{y}; a = y^{(1/-N)} = y^{-(1/N)} = 1/\sqrt[N]{y}$$

The LL and LL0 scales mated; Z on LL = 1/Z on LL0, Z on LL0 = 1/Z on LL

The solution scales depend on which C scale index, left or right, is used

$a = y^{1/N} = \sqrt[N]{y}; a = y^{(1/-N)} = 1/\sqrt[N]{y}$ 1) move cursor to y on LLy/LL0y scale 2) move slide; N on C scale under cursor 3) move cursor to C index 4) determine the proper LLa/LL0a scale as described below 5) read $a = y^{1/N} = \sqrt[N]{y}$ under cursor on proper LLa/LL0a scale	
LL/LL0 Solution Scales $y > 1$ $N > 0$; y on LLy scale, read a on LLa scale $N < 0$; y on LLy scale, read a on LL0a scale $0 < y < 1$ $N > 0$; y on LL0y scale, read a on LL0a scale $N < 0$; y on LL0y scale, read a on LLa scale	
LLa/LL0a solution scales Write N as $N.mnnE^{(+/-)x}$ ($E^{(+/-)x} = 10^{(+/-)x}$) $N = 51.3 = 5.13E^1, x = 1; N = 0.00329 = 3.29E^{-3}, x = -3$	
Left C index over a on LLa/LL0a scale knowing $N^{(+/-)x}$ LLa/LL0a solution scale = LLy/LL0y scale - x	Right C index over a on LLa/LL0a scale knowing $N^{(+/-)x}$ LLa/LL0a solution scale = LLy/LL0y scale - x - 1
$a = y^{(1/N)} = \sqrt[N]{y}$ $y > 1$ on LLy scale; $N > 0$ (positive) on C scale; $a > 1$ on LLa scale	
let y = 20 (LL3 scale); $N = 1.5128E^2, x = 2$ LLa scale = LLy scale - x = 3 - 2 = 1 (LL1 scale) a (LL1) = $y^{(1/N)} = (20)^{(1/1.5128E2)} \approx 1.02$ (1.02000)	let y = 1.0114 (LL1 scale); $N = 7.1447E^{-3}, x = -3$ LLa scale = LLy scale - x - 1 = 1 - (-3) - 1 = 3 (LL3 scale) a (LL3) = $y^{(1/N)} = (1.01140)^{(1/7.1447E-3)} \approx 4.89$ (4.88619)
$a = y^{-(1/N)} = 1/y^{(1/N)} = 1/\sqrt[N]{y}$ $y > 1$ on LLy scale; $N < 0$ (negative) on C scale; $0 < a < 1$ on LL0a scale	
let y = 20 (LL3 scale); $N = -1.5128E^2, x = 2$ LL0a scale = LLy scale - x = 3 - 2 = 1 (LL01 scale) a (LL01) = $y^{-(1/N)} = (20)^{-1/1.5128E2} \approx 0.9804$ (0.98039)	let y = 1.0114 (LL1 scale); $N = -7.1447E^{-3}, x = -3$ LL0a scale = LL0y scale - x - 1 = 1 - (-3) - 1 = 3 (LL03 scale) a (LL03) = $y^{-(1/N)} = (1.0114)^{-1/7.1447E-3} \approx 0.2045$ (0.20463)
$a = y^{(1/N)} = \sqrt[N]{y}$ $0 < y < 1$ on LL0y scale ; $N > 0$ (positive) on C scale; $0 < a < 1$ on LL0a scale	
let y = 0.05000 (LL03 scale); $N = 1.5128E^2, x = 2$ LL0a scale = LL0y scale - x = 3 - 2 = 1 (LL01 scale) a (LL01) = $y^{(1/N)} = (0.050)^{(1/1.5128E2)} \approx 0.9804$ (0.98039)	let y = 0.98873 (LL01 scale); $N = 7.1447E^{-3}, x = -3$ LL0a scale = LL0y scale - x - 1 = 1 - (-3) - 1 = 3 (LL03 scale) a (LL03) = $y^{(1/N)} = (0.98873)^{(1/7.1447E-3)} \approx 0.2045$ (0.20467)
$a = y^{-(1/N)} = 1/y^{(1/N)} = 1/\sqrt[N]{y}$ $0 < y < 1$ on LL0y scale ; $N < 0$ (negative) on C scale; $a > 1$ on LLa scale	
let y = 0.05000 (LL03 scale); $N = -1.5128E^2, x = 2$ LLa scale = LL0y scale - x = 3 - 2 = 1 (LL1 scale) a (LL1) = $y^{-(1/N)} = (0.050)^{-1/1.5128E2} \approx 1.02$ (1.02000)	let y = 0.98873 (LL01 scale); $N = -7.1447E^{-3}, x = -3$ LLa scale = LL0y scale - x - 1 = 1 - (-3) - 1 = 3 (LL3 scale) a (LL3) = $y^{-(1/N)} = (0.98873)^{-1/7.1447E-3} \approx 4.89$ (4.88619)

XI

The Log-Log Scales

$$N = \text{Log}_a y$$

The solution scales depend on which C scale index, left or right, is used

<p>$N = \text{Log}_a y$</p> <p>1) move cursor to a on LLa/LL0a scale 2) move slide; C index under cursor 3) move cursor to y on LLy/LL0y scale 4) determine $NE^{(+/-)x}$, N positive or negative, and exponent, $(+/-)x$, as described below 5) read $N = \text{Log}_a y$ under cursor on C scale</p>	
<p>LLa/LL0a and LLa/LL0y scales</p> <p style="text-align: center;">$a > 1$</p> <p>$y > 1$; a on LLa scale, y on LLy scale, read positive N on C scale $0 < y < 1$; a on LLa scale, y on LL0y scale, read negative N on C scale</p> <p style="text-align: center;">$0 < a < 1$</p> <p>$y > 1$; a on LL0a scale, y on LLy scale, read negative N on C scale $0 < y < 1$; a on LL0a scale, y on LL0y scale, read positive N on C scale</p>	
<p>NE^x on the C scale</p> <p>Note $E^{(+/-)x} = 10^{(+/-)x}$</p>	
<p>Left C index over a on LLa/LL0a scale $x = \text{LLy/LL0y scale} - \text{LLa/LL0a scale}$</p>	<p>Right C index over a on LLa/LL0a scale $x = \text{LLy/LL0y scale} - \text{LLa/LL0a scale} - 1$</p>
<p><i>a > 1 on LLa scale; y > 1 on LLy scale; N > 0 (positive) on C scale</i></p>	
<p>Let a = 1.02 (LL1 scale) and y = 20 (LL3 scale) $x = \text{LLy scale} - \text{LLa scale} = 3 - 1 = 2$; NE^2 $N = \text{Log}_{(1.02)}(20) \approx 1.515E^2 (15128E^2)$</p>	<p>Let a = 7 (LL3 scale) and y = 1.014 (LL1 scale) $x = \text{LLy scale} - \text{LLa scale} - 1 = 1 - 3 - 1 = -3$; NE^{-3} $N = \text{Log}_7(1.014) \approx 7.15E^{-3} (7.1447E^{-3})$</p>
<p><i>a > 1 on LLa scale; 0 < y < 1 on LL0y scale; N < 0 (negative) on C scale</i></p>	
<p>Let a = 1.02 (LL1 scale) and y = 0.05 (LL03 scale) $x = \text{LL0y scale} - \text{LLa scale} = 3 - 1 = 2$; NE^2 $N = \text{Log}_{(1.02)}(0.05) \approx -1.515E^2 (-15128E^2)$</p>	<p>Let a = 7 (LL3 scale) and y = 0.98619 (LL01 scale) $x = \text{LL0y scale} - \text{LLa scale} - 1 = 1 - 3 - 1 = -3$; NE^{-3} $N = \text{Log}_7(0.98619) \approx -7.15E^{-3} (-7.1447E^{-3})$</p>
<p><i>0 < a < 1 on LL0a scale; y > 1 on LLy scale; N < 0 (negative) on C scale</i></p>	
<p>Let a = 0.9804 (LL01 scale) and y = 20 (LL3 scale) $x = \text{LLy scale} - \text{LL0a scale} = 3 - 1 = 2$; NE^2 $N = \text{Log}_{(0.9804)}(20) \approx -1.515E^2 (-15128E^2)$</p>	<p>Let a = 0.14286 (LL03 scale) and y = 1.014 (LL1 scale) $x = \text{LLy scale} - \text{LL0a scale} - 1 = 1 - 3 - 1 = -3$; NE^{-3} $N = \text{Log}_{(0.14286)}(1.014) \approx -7.15E^{-3} (-7.1447E^{-3})$</p>
<p><i>0 < a < 1 on LL0a scale; 0 < y < 1 on LL0y scale; N > 0 (positive) on C scale</i></p>	
<p>Let a = 0.9804 (LL01 scale) and y = 0.05 (LL03 scale) $x = \text{LL0y scale} - \text{LL0a scale} = 3 - 1 = 2$; NE^2 $N = \text{Log}_{(0.9804)}(0.05) \approx 1.515E^2 (1.5128E^2)$</p>	<p>Let a = 0.14286 (LL03 scale) and y = 0.98619 (LL01 scale) $x = \text{LL0y scale} - \text{LL0a scale} - 1 = 1 - 3 - 1 = -3$; NE^{-3} $N = \text{Log}_{(0.14286)}(0.98619) \approx 7.15E^{-3} (7.1447E^{-3})$</p>