The following is JOS Plus supplemental information for the article:
The Hemmi 301 and the Boonshaft & Fuchs Slide Rules
Richard Smith Hughes
in JOS 20:2

The Hemmi 301 Control Engineering Frequency Response Slide Rule
Richard Smith Hughes

The following scanned manual is courtesy of Paul Ross.

The table below will aid in the normalization of the transfer function

I	Write τ as $X.xxE^{z}$
II	Forget the exponent E^{z}, $\tau_{Normalized} = X.xx$
III	Normalize ω with respect to τ; $\omega_{Normalized} = \omega / E^{z}$
IV	Place cursors over the various $\tau_{Normalized}$ (neglecting the exponent, E^{z})
V	Move the diamond, at 45 on the θ scale, under the wanted $\omega_{Normalized} = \omega / E^{z}$ a) Read $\theta\tau$ on the θ scale ($\theta\tau = \mathrm{Tan}^{-1}\ \omega\tau$) b) Read the magnitude, in dBV, on the g scale ($g = 20\mathrm{Log}(1 + s\tau) = 20\ \mathrm{Log}(1 + j\ \omega\tau) = 20\ \mathrm{Log} \sqrt{[1 + (\omega\tau)^2]}$ dBV c) Read the normalized frequency on the back side under the hairline ($f_{Normalized} = \omega / 2\pi$); multiply by E^{z} for the frequency d) Calculate the total magnitude, dBV_{Total} e) Calculate the total phase angle, θ_{Total}

INSTRUCTION MANUAL

FOR

CONTROL ENGINEERING SLIDE RULE

FOR

FREQUENCY RESPONSE COMPUTATION

Cat. No. 301

No. 301A (With Attachment)

by

Keisuke IZAWA

Formerly Assistant Professor of

TOKYO INSTITUTE OF TECHNOLOGY

Presently Associated Professor at

PURDUE UNIVERSITY

SUN

HEMMI Bamboo Slide Rule Mfg, Co., Ltd.

Tokyo, Japan

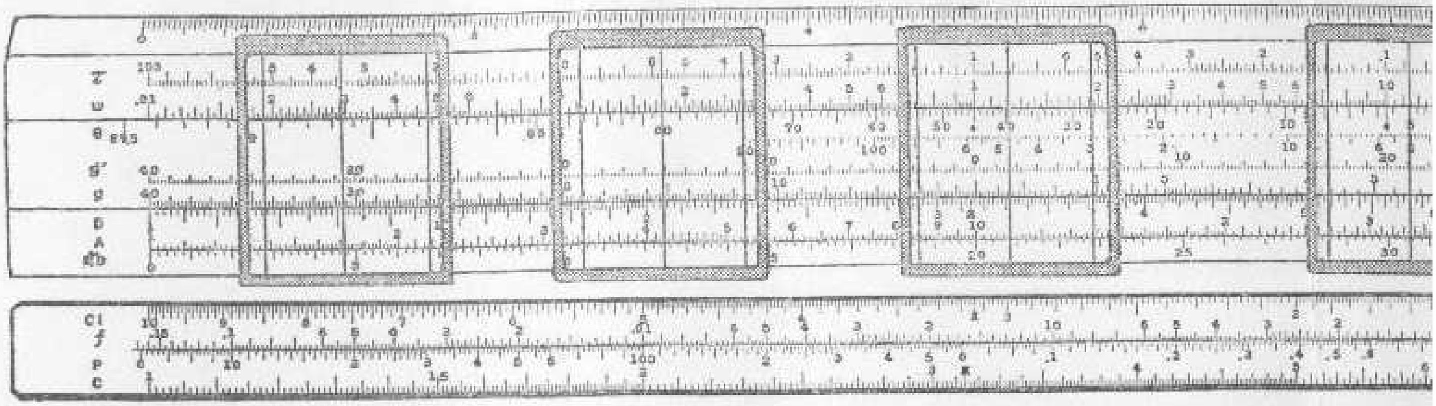

Fig. 1 Slide Rule for Frequency Response Cat. No. 301

INTRODUCTION

Frequently in the field of automatic control engineering, mechanical engineering, electrical engineering, communication engineering, e.t.c., it is necessary to compute the magnitude ratios and phase shift angles for physical systems subjected to sinusoidal inputs. This slide rule has been invented by Professor Keisuke IZAWA of Tokyo Institute of Technology firstly in order to obtain the frequency responses of physical systems where the transfer functions of the systems can be broken into real linear and exponential factors.

The general transfer function of the above will be represented by $G(s)$:

$$G(s)=\frac{Ks^{h}\ (T_{a1}s+1)\ (T_{a2}s+1)\cdots\cdots\cdots\cdots(T_{am}s+1)}{(T_{b1}s+1)\ (T_{b2}s+1)\cdots\cdots\cdots\cdots(T_{bn}s+1)}\ e^{-Ls} \qquad (1)$$

where K is a constant, h is an integer, the quantities T_{a1}, T_{a2}, ………, are lead time constants of the system, while T_{b1}, T_{b2}, ………, are lag time constants, and L is the dead time.

The slide rule will be utilized not only in automatic control problems but also in other problems discussing the frequency response characteristics, such as in mechanical or electrical engineering fields.

COMPOSITION OF THE SLIDE RULE

The slide rule is of the Mannheim type and consists of the following functional scales and cursors, as shown in Figure 1.

Front Surface

a) τ and ω scales on the upper fixed rule. b) D, A, and db scales on the lower fixed rule.

c) θ, θ_d, g and g' scales as well as reference marks ◆, ▼, and ▲, on the slide.

Rear Surface

a) CI, f, P, and C scales on the slide.

b) Reference hairlines in the windows at both ends of the middle section.

The slide rule shown in Figure 1 has four cursors with three hairlines each.

SLIDE RULE CALCULATIONS

1. Fundamental Operations

1.1 The transfer function $G(s)$ of a system S_1 given by

$$G(s) = (0.5s + 1) \quad = \frac{2}{s+2}$$

illustrates a system of lead characteristics. Assume that the magnitude ratio g in db and the phase angle θ in degree are desired at $\omega = 9$ radians per second.

The steps in obtaining the magnitude ratio and phase angle are as follows:

1) Set the cursor hairline over $T = 0.5$ on the τ scale.

2) Set the diamond ◆ directly below 9 on ω scale.

3) Read the magnitude ratio $g = 13.3$ db in db under the hairline on the g scale. And read the phase angle $\theta = 77.4°$ under the same hairline on the θ scale. (see Fig. 2)

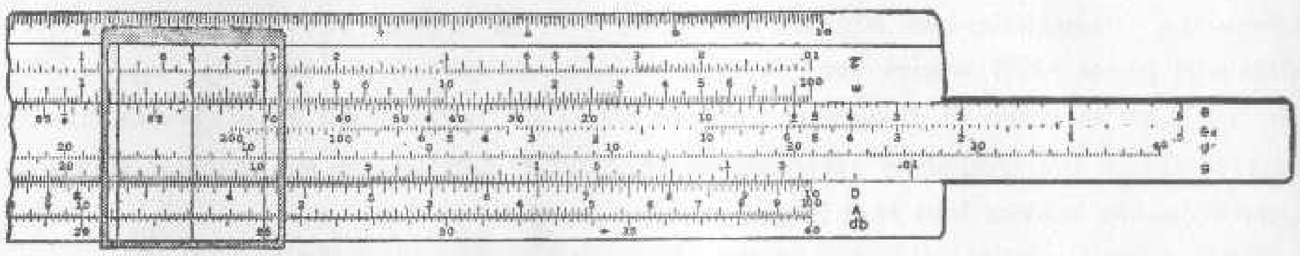

Fig. 2

1.2 As an example, consider the system S_2 with transfer function $(0.5s + 1)^{-1}$. We wish the magnitude ratio and phase angle for S_2 when $\omega = 9$. From results of Fig. 2 for the system S_1 with $T = 0.5$ and $\omega = 9$, we find that

the magnitude ratio $= -13.3$ db and the phase angle $= -77.4°$

1.3 The computation of the phase angle with the slide rule for a system having a transfer function Ks^h is not necessary since the phase may be determined mentally as follows. A system with a differentiating factor Ks^h, where s^h represents the hth derivatives, will have phase shift of 90h°, i.e, 90h degrees. A system with an integrating factor K/s^h, where $1/s^h$ represents hth integrations, will have a phase shift of $-90h°$.

Computation of the magnitude ratio g' in db for an integrating or differentiating element may be handled quickly through the use of the slide rule. The ω and the g' scales are used to compute the magnitude ratio of the integration element. The g' scale is a linearly spaced scale, with 40 on the left, zero at the midpoint and 40 on the right.

For example let us determine the magnitude ratio for a system S_3 having a transfer function $30/s$. for $\omega = 4$ radians per minute. (K is 30 radians per minute). The following steps may be taken for the computation of the magnitude ratio of the system S_3. (Fig. 3)

1) Set the cursor hairline over $K = 30$ on the ω scale.

2) Set the diamond ◆ directly below 4 on the ω scale.

3) Read the magnitude ratio $g' = 17.5$ db under the hairline on the g' scale. Note black figures be read as positive while red ones as negative, for any integral elements.

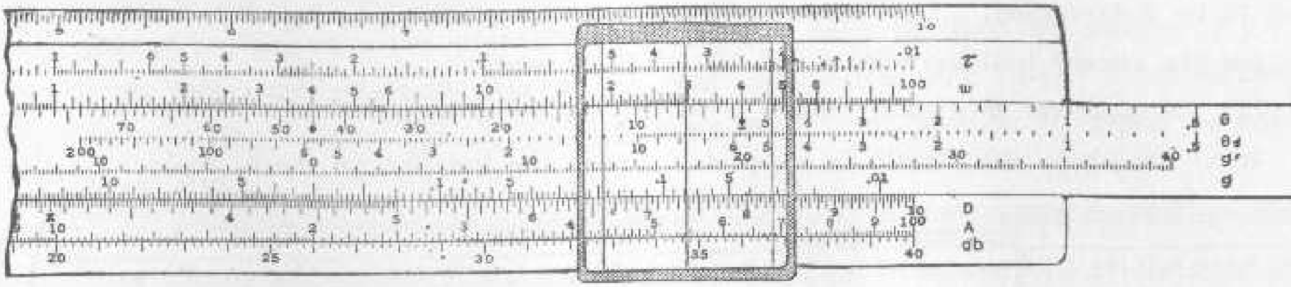

Fig. 3

1.4 If the magnitude ratio of the system S_4 with transfer function $0.1\,s$ is wanted to be determined when $\omega = 4$, we carry the following steps.

1) Set cursor hairline over $K = 0.1$ on the τ scale.
2) Set the diamond ◆ directly below $\omega = 4$ on the ω scale.
3) Read the magnitude ratio as -8 db under the hairline on the g' scale. Note black figures be read as negative while red ones as positive for any differential elements.

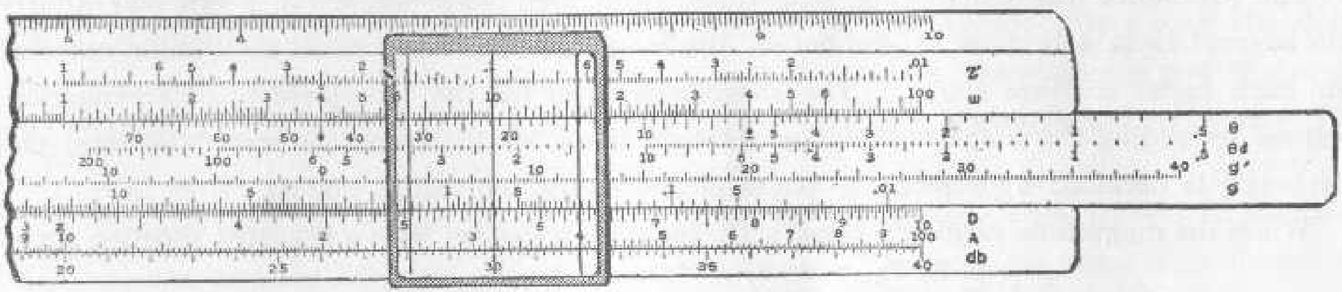

Fig. 4

1.5 Consider the system S_5 with transfer function $20/s^2$. We wish the magnitude ratio g' for S_5 when $\omega = 3$. The steps in obtaining the magnitude ratio are as follows.

1) Set the cursor hairlines over $K_1 = 1$ and $K_2 = 20$ on the ω scale. $(K = K_1\ K_2)$
2) Set the diamond ◆ directly below $\omega = 3$ on the ω scale.
3) Read the magnitude ratios $g_1' = -9.6$ db and $g_2' = 16.5$ db under the each hairline on the g' scale.
4) The magnitude ratio wanted to be determined is an algebraic sum of g_1' and g_2':

$$g' = g_1' + g_2' = -9.6 + 16.5 = 6.9 \text{ db.}$$

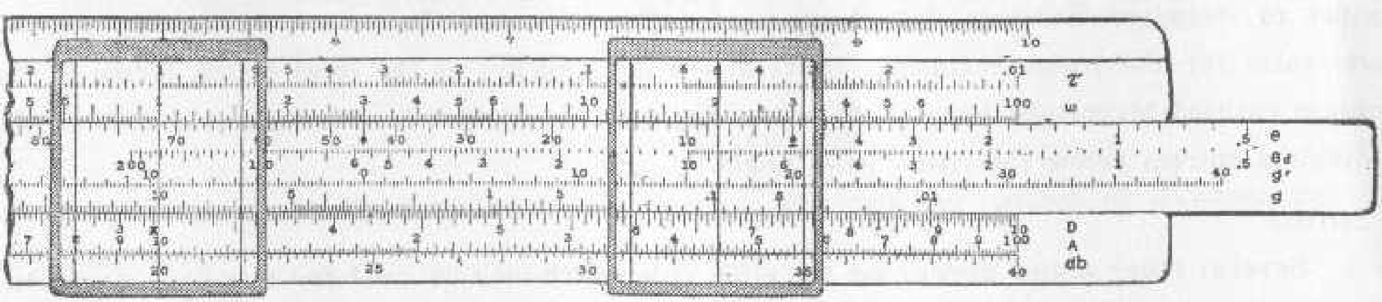

Fig. 5

1.6 The dead time factor e^{-Ls} in equation (1) remaines to be covered. We assume that we have a system S_6 with the transfer function $F(s)$, where $\quad F(s) = e^{-Ls}$

The magnitude ratio for the system S_6 is always unity or 0 db no matter what values of s and L may be concerned. Therefore only phase computations need be considered.

As an example we consider the system S_6 has $L = 0.5$ and phase shift for S_6 at $\omega = 0.5$

is going to be determined. The following steps will be taken as in Fig. 6.

1) Set the cursor hairline over L = 0.5 on the τ scale.
2) Set the diamond ◆ directly below ω = 0.5 on the ω scale.
3) Read the phase shift angle $\theta_d = -14.3^\circ$ under the hairline on the θ_d scale.

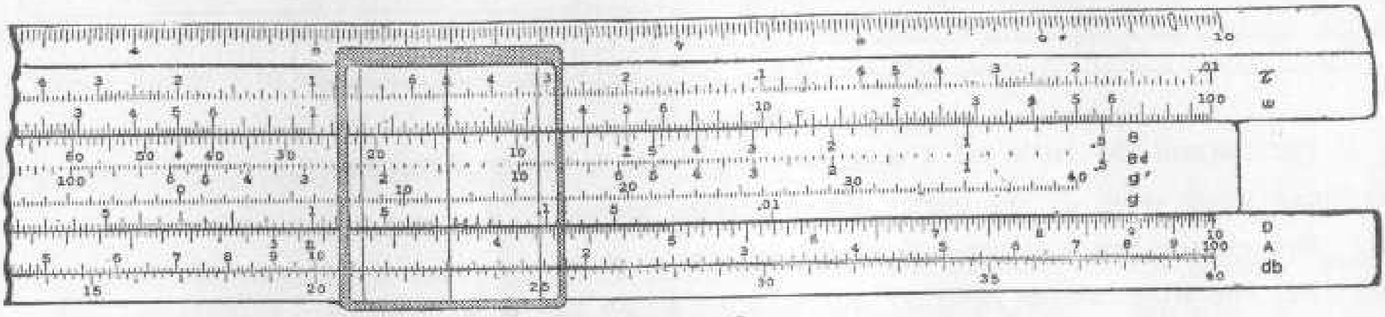

Fig. 6

2. Calculation for Transfer Functions with Several Factors.

The procedure for calculating magnitude ratio and phase angle for a transfer function with several factors is done by computing the magnitude ratio and phase shift angle associated with each factor as done above. The magnitude ratio for the complete transfer function is obtained by adding the magnitude ratios obtained in db for individual factors. The total phase shift angle is obtained by adding the phase angles of the individual factors.

While the magnitude ratio and phase shift angle for a system with a transfer function such as

$$F(s) = \frac{K(T_{a1}\, s + 1)\, e^{-Ls}}{s\,(T_{b1}\, s + 1)} \qquad (2)$$

is being computed, it is convenient to form the following chart.

Factor	Magnitude Ratio	Phase Angle
K/s	....	−90
$(T_{a1}\, s + 1)$	....	....
$(T_{b1}\, s + 1)$	....	....
e^{-Ls}	0	....
Total	....	....

The entries below the headings Magnitude Ratio and Phase Angle respectively correspond to values of magnitude ratio in db and phase in degrees for each factor of (2). The magnitude ratio for the transfer function $F(s)$ at $\omega = \omega_a$ appears as the total of the terms in the column entitled Magnitude Ratio. The phase angle of the system at $\omega = \omega_a$ is the sum of the individual entries below the title Phase Angle.

Note.

N-1 Several other scales appear on the slide rule which may be used for auxiliary computations. These other scales are the C, CI, D, A and db scales. Their usefulness is more apparent when the slide is turned over. The C and D scales are employed for multiplication and division. The CI scale is the reciprocal of the C scale. The A scale is graduated such that values on the A scale are the square of the entries directly above on the D scale. The db scale and A scales serve as a means to convert actual magnitude ratio to magnitude ratio in db.

N-2 All of the computations to this have originated with ω. When the frequency $f = f_a$ is known instead of ω the preceding computations may be done by placing $f = f_a$ on the f scale below the reference line on the rear side of the slide rule. This locates the diamond directly below the ω_a corresponding to f_a, the position it would be set if the ω_a were given.

N-3 When the period $P = P_a$ is given instead of ω_a or f_a, the same operations may be carried out by placing $P = P_a$ on the P scale below the reference line of the rear side of the slide rule.

N-4 Each of the cursors has three hairlines spaced by equal distance, lending to a convenient means of converting one value of f, P, or ω to a value of another. We set $\omega = \omega_a$ under the left hairline on the ω scale. The frequency f_a corresponding to ω_a is then gotten by dividing the entry, that appears under the center hairline, by ten. The figure under the same hairline on the τ scale is 10 th times of period P_a corresponding to ω_a. The inverse operation, converting f or P to ω, is accomplished in the same manner.

N-5 Two other reference marks $\overline{\blacklozenge}$ and $\underline{\blacklozenge}$ appear on the θ scale along with the diamond. When the diamond $\blacklozenge$ is placed at $\omega = \omega_a$ on the ω scale, the marks $\overline{\blacklozenge}$ and $\underline{\blacklozenge}$ are located below $\omega = \omega_a/10$ and $\omega = 10\,\omega_a$ respectively. Consider a system with a transfer function $G(s)$ given by $\quad G(s) = 1/(0.04\,s + 1)$.

Assume that we desire the magnitude ratio for the system when $\omega_a = 500$. Immediately we see that the diamond cannot be located below $\omega_a = 500$ since the ω scale covers entries within the interval $0.01 \leqq \omega \leqq 100$. The alternate procedure we follow is to place the mark $\overline{\blacklozenge}$ below $\omega_a/10$, the entry 50 on the ω scale and proceed with the calculation as previously illustrated. We then obtain -26 db as the magnitude ratio. (Fig. 7).

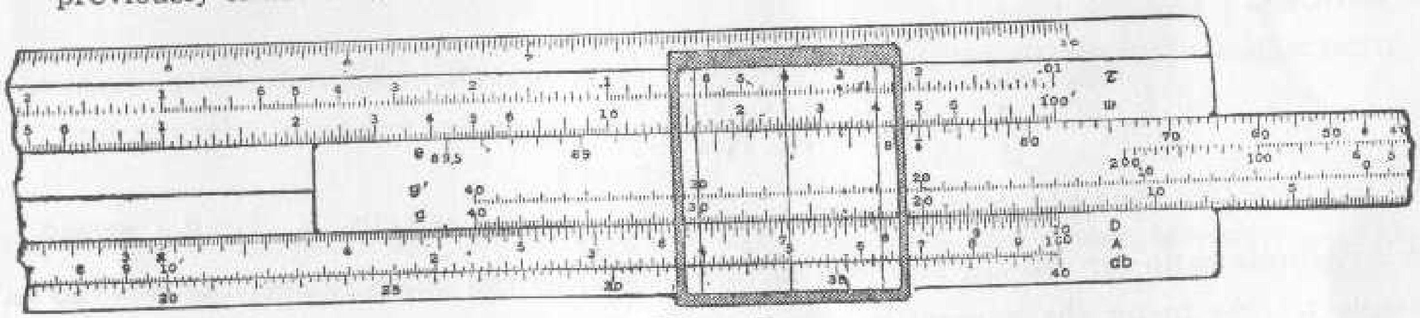

Fig. 7

EXAMPLES

The following three examples will further illustrate the method of computation with the slide rule.

Example-1

Determine the magnitude ratio and phase angle for a system with transfer function

$$G(s) = K\frac{T_a\,s + 1}{s(T_1\,s+1)\ (T_2\,s+1)}$$

where $\quad \omega = 4$ radians per minute, $\quad K = 30, \quad T_a = 1$ minute,

$T_1 = 8$ minutes, $\quad T_2 = 0.2$ minute, The first step is to set cursor hairlines over K=30 on the ω scale, $T_a=1$, $T_1=8$, and T_2 = 0.2 on the τ scale. The cursor setting is now accomplished and it is ready for obtaining magnitude ratios and phase angles at any ω. The next step is to set the diamond below the value ω = 4 on the ω scale. (Fig. 8) The magnitude ratio of the factor $30/s$ is then read on the g' scale under the corresponding hairline. For each of the remaining factors corresponding to two lags and a lead, on the g scale the magnitude ratio in db of each element is read under the corresponding hairline, while its phase angle will appear under the same hairline on the θ scale.

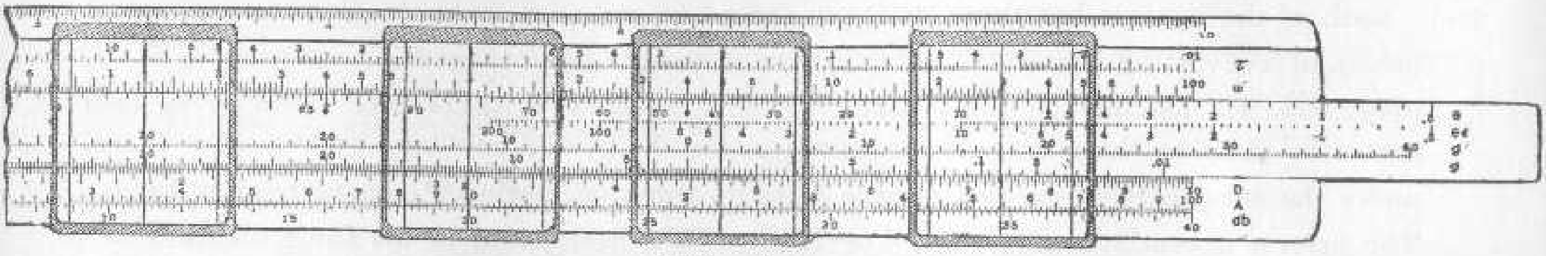

Fig. 8

The following chart will list each reading obtained from the above operations.

Factor	Magnitude Ratio	Phase Angle
$30/s$	17.5	−90
$(s + 1)$	12.3	76
$(8s + 1)$	−30.1	−88.2
$(0.2s + 1)$	−2.2	−38.7
Total	−2.5	−140.9

Hence the magnitude ratio in decibels of G(s) is −2.5 db while the phase shift angle is −141°.

Example-2

Determine the magnitude ratio and phase angle for a system with the transfer function

$$G(s) = K \frac{(T_a\, s + 1)}{s^2\,(T_1\, s + 1)}$$

for $\omega=4$ radians per second where K=30, $T_a=8$ seconds, $T_1=0.2$ second.

The magnitude ratio for the lag and lead factors as well as the phase remain the same as in Example-1. We factor the integration term as follows,

$$30/s^2 = (30/s)\ (1/s)$$

A chart is then constructed in the same manner as done in Example-1. (see also Fig. 8)

Factor	Magnitude Ratio	Phase Angle
$30/s$	17.5	−90
$1/s$	−12	−90
$(8s + 1)$	30.1	88.2
$(0.2s + 1)$	−2.2	−38.7
Total	33.4	−130.5

Hence for this system with the transfer function given above we have a magnitude ratio of 33.4 db and a phase lag of −131° at ω = 4 radians per second.

Example-3

Determine the magnitude ratio and phase angle for a system with the transfer function

$$G(s) = K\,\frac{(T_a s + 1)\,e^{-Ls}}{s(T_1 s + 1)}$$

when the input frequency is 0.5 cycle per minute or its period is 2 minutes. The constants of the transfer function are given to be

$K = 30$, $T_a = 8$ minutes, $T_1 = 1$ minute, and $L = 0.2$ minute.

The first step is to set each cursor hairline over $K = 30$ on the ω scale and $T_a = 8$, $T_1 = 1$, and $L = 0.2$ on the τ scale. The next step is to convert given $f = 0.5$ to ω by placing the entry of 0.5 of the f scale at the reference line on the rear side. The diamond is now located below the value 3.14 of ω scale corresponding to $f = 0.5$ and in the correct position for completing the computation. (Fig. 9)

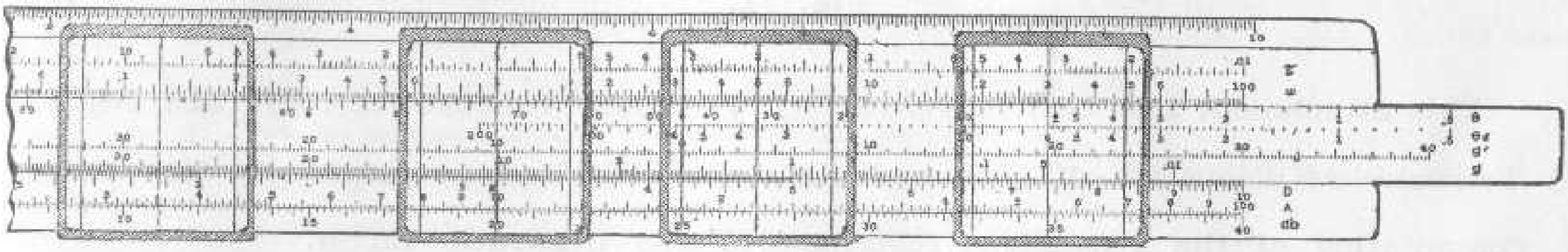

Fig. 9-1

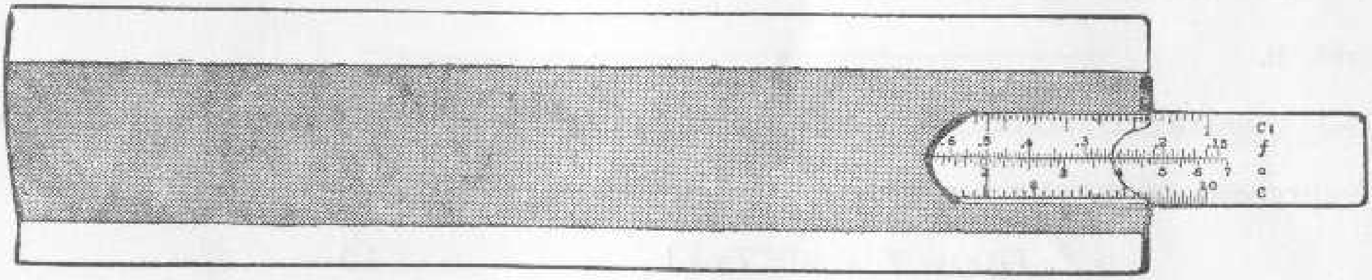

Fig. 9-2

After fundamental operation and reading for each factor in the above transfer function, we have the following chart.

Factor	Magnitude	Phase Angle
$30/s$	19.6	−90
$(8s + 1)$	28	87.7
$(s + 1)$	−10.3	−72.3
e^{-Ls}	0	−36
Total	37.3	−110.6

From the totals it is seen that the magnitude ratio is 37.3 db and the phase angle is −111°.

HOW TO USE

THE ATTACHMENT OF THE FREQUENCY RESPONSE SLIDE RULE (No. 301A)

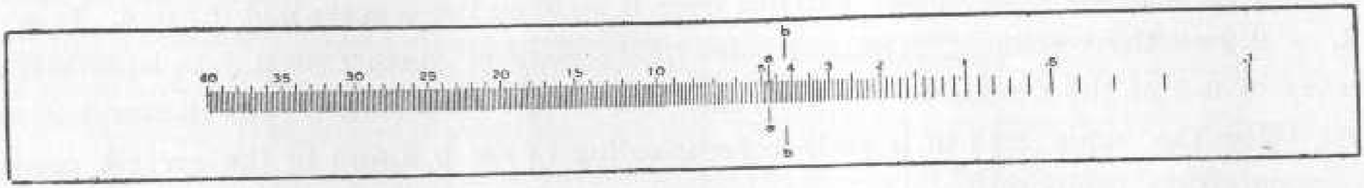

Fig. 1.

This attachment has been invented by Professor Keisuke IZAWA and Mr. Tomoaki MORINAGA of Tokyo Institute of Technology especially for the computation of the frequency responese of the quadratic factor.

The attachment consists of one functional scale (Fig 1) and two reference lines **a** and **b**.

The quadratic factor of a minimum phase system fransfer function will be represented in the form as

$$G(s) = T^2s^2 + 2\zeta Ts + 1 \qquad (1)$$

where T is a time constant, and ζ is a relative damping.

In order to explain the method of operation of a slide rule, diagram under the following assumption is used.

↓ Set indicator at here

↗ ↖ Set the figure of the slide or attachment at here.

* The value is found here.

The numbers in the circles ○ indicate the process of operation.

CALCULATIONS

1. Fundamental operations

1.1 Consider the transfer function $G(s)$ given by

$$G(s)=0.16s^2+0.48s+1$$

comparing this with Eq. (1), we obtain $T=0.4$ sec. and $\zeta=0.6$. Assume that the magnitude ratio g_2 in db and the phase angle θ_2 in degree are desired at $\omega=2$ radian per second. The steps in obtaining g_2 and θ_2 are as follows:

1) Set the cursor hairline over $T=0.4$ on the τ scale.
2) Set the diamond ◆ below $\omega=2$ on the ω scale.
3) Read the parameter $g'=1.9$ on the g' scale under the cursor hairline.
4) Then, insert the attachment into the gap between slide rule surface and cursor, and set $g'=1.9$ on the attachment scale just over $100\zeta=60$ on the θ_d scale.
5) Read the phase angle $\theta=69.5°$ under the reference line **a** on the θ scale. At the same time, read $g_{2a}=9.2$ db on the g scale under the same line.
6) Set the reference line **b** under the cursor hairline.
7) Read $g_{2b}=9.0$ db on the g' scale under $g'=1.9$ on the attachment scale. Then, the magnitude ratio is $g_2=g_{2a}-g_{2b}=9.2-9.0=0.2$ db.

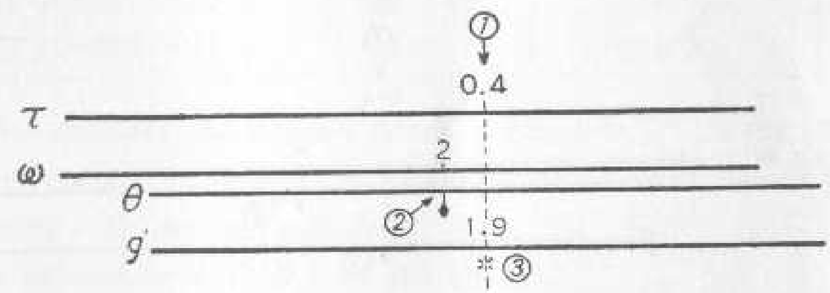

Fig. 2-1

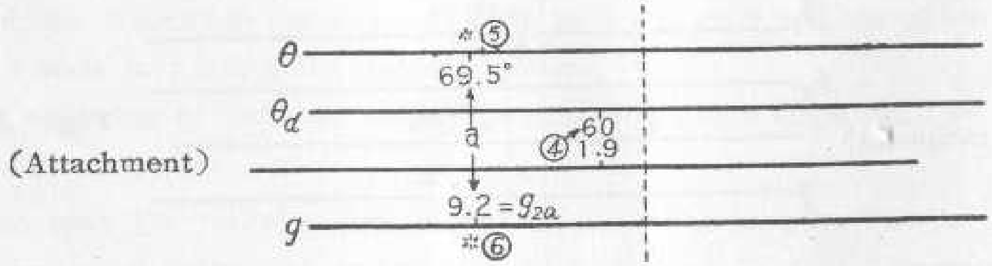

Fig. 2-2

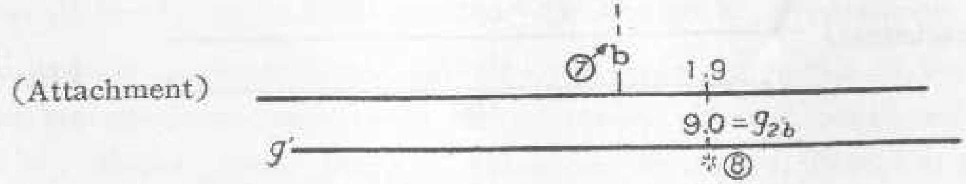

Fig. 2-3

It is convenient to list above obtained values in the following form.

$$\begin{array}{rl} & g_{2a}=9.2 \\ -) & g_{2b}=9.0 \\ \hline & g_2=0.2\ \text{db} \end{array} \qquad \theta_2=69.5°$$

1.2 The graduated number on the left half of the g' scale is coloured red. This means negative of g'. When the parameter g' is obtained negatively, we must be careful in calculation of the phase angle. In that case, the phase angle is the supplementary angle of the value obtained on the θ scale.

As an example, assume that the magnitude ratio and the phase angle of $(0.16s^2+0.48s+1)$ are desired at $\omega=4$ radian per second. The calculating steps are as follows:

1) Set the cursor hairline over $T=0.4$ on the τ scale.
2) Set the diamond ◆ below $\omega=4$ on the ω scale.
3) Read the parmeter $g'=-4.1$ on the g' scale under the cursor hairline. This value is negative.
4) Set $|g'|=4.1$ on the attachment scale just over $100\zeta=60$ on the θ_d scale.
5) Read 51° under the reference line **a** on the θ scale. Therefore, the phase angle is $\theta_2=180°-51°=129°$ At the same time, read $g_{2a}=4.0$ db on the g scale under the same line.
6) Set the reference line **b** under the cursor hairline.
7) Read $g_{2b}=-3.9$ on the g' scale under $|g'|=4.1$ on the attachment scale. Then, the magnitude ration is $g_2=4.0-(-3.9)=7.9$ db.

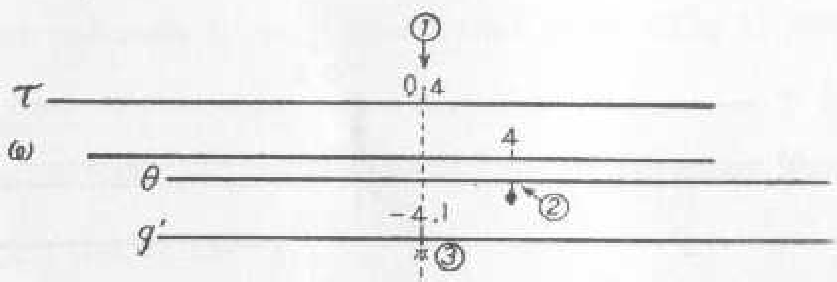

Fig. 3-1

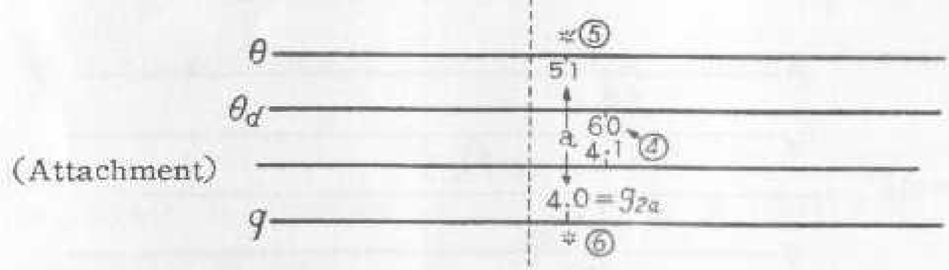

Fig. 3-2

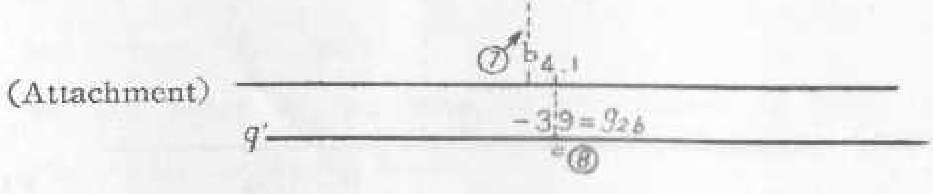

Fig. 3-3

$$\begin{array}{rl} g_{2a}= & 4.0 \\ -)\,g_{2b}= & -3.9 \\ \hline g_2= & 7.9\text{db} \end{array}$$

$\theta_2=180°-51°=129°$

1.3 For the transfer function $(0.16s^2+0.48s+1)^{-1}$ at $\omega=2$ radian per second, the magnitude ration and the phase angle are -0.2 db and $-69.5°$, respectively. This is easily understood for the results of 1.1.

1.4 When $\zeta \geqq 1$, the quadratic factor can be factorized into two real linear factors. Hence, in this case, magnitude ratio and phase angle can be calculated only by the slide rule without the attachment. But these values can be calculated directly by the attachment and the slide rule without factorization. Calculating method is entirely the same as that in case of $\zeta<1$.

2. Another application

This attachment and the slide rule are also available for such non-minimum phase systems as $(1-Ts)$, $(Ts-1)$ and $(T^2s^2-\zeta Ts+1)$. The following table presents their magnitude ratios and phase angles in comparison with their corresponding minimum phase systems.

Table 1

Factor	Magnitude ratio	Phase angle
$(1+Ts)$	g_1 db	$\theta_1°$
$(1-Ts)$	g_1	$-\theta_1°$
$(Ts-1)$	g_1	$180°-\theta_1°$
$(T^2s^2+2\zeta Ts+1)$	g_2 db	$\theta_2°$
$(T^2s^2-2\zeta Ts+1)$	g_2	$-\theta_2°$

2.1 As an example, we consider the transfer function $(1-0.5s)$. To find its magnitude ratio and phase angle at $\omega=9$ radian per second, you must calculate these values for $(1+0.5s)$ in the first step. The calculating steps are as follows:

1) Set the cursor hairline over $T=0.5$ on the τ scale.
2) Set the diamond ◆ below $\omega=9$ on the ω scale.
3) Read the magnitude ratio $g_1=13.3$ db on the g scale and the phase angle $\theta_1=77.4°$ on the θ scale both under the cursor hairline.

Then, according to Table 1, magnitude ratio and phase angle for $(1-0.5s)$ are 13.3 db and $-77.4°$, respectively.

2.2 Consider next the transfer function $(0.5s-1)$. The magnitude ratio and the phase angle for $(1+0.5s)$ at $\omega=9$ radian per second are 13.3 db and $77.4°$. Therefore, from Table 1, these values for $(0.5s-1)$ are 13.3 db and $180°-77.4°=102.6°$, respectively.

2.3 As an example of quadratic factor, we take the transfer function $(0.16s^2-0.48s+1)$. The magnitude ratio and the phase angle of $(0.16s^2+0.48s+1)$ at $\omega=2$ radian per second are $g_2=0.2$ db and $\theta_2=69.5°$. Hence, from Table 1, values for $(0.16s^2-0.48s+1)$ at the same frequency are 0.2 db and $-69.5°$, respectively.