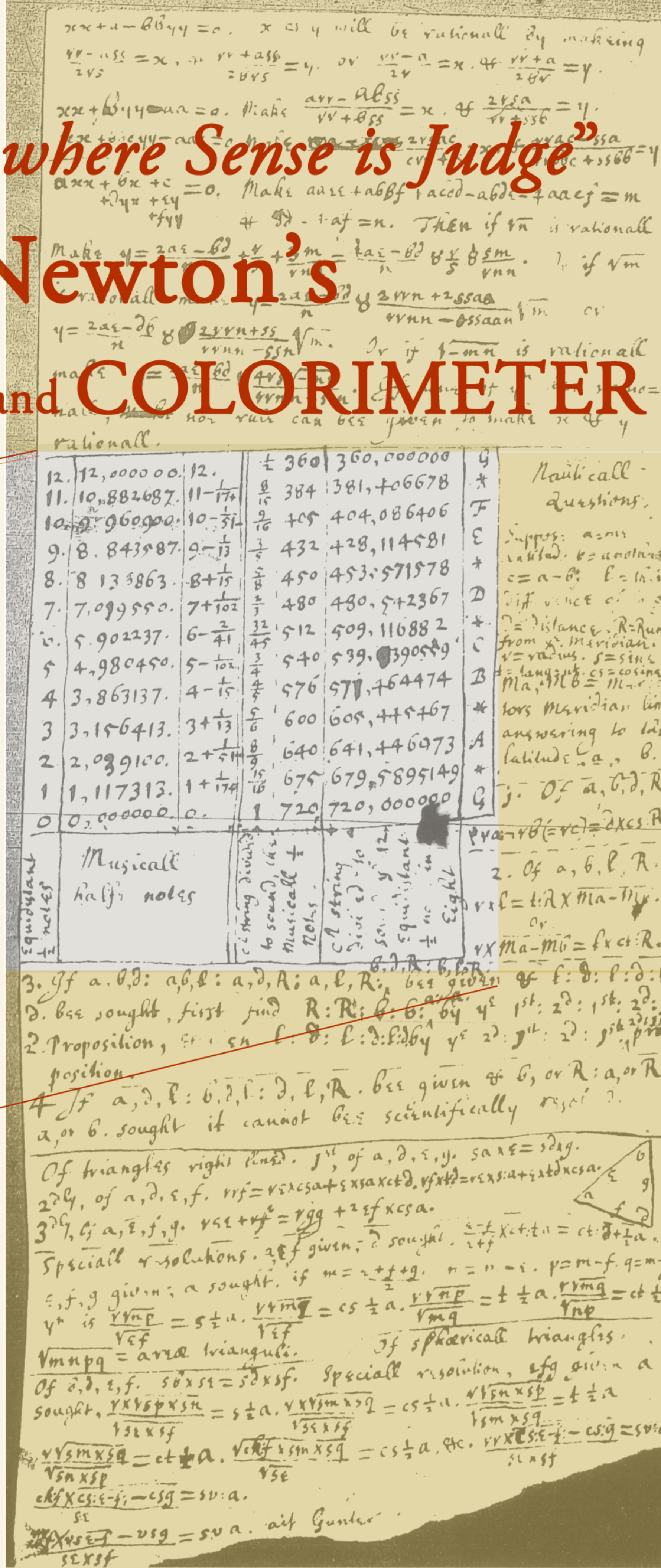
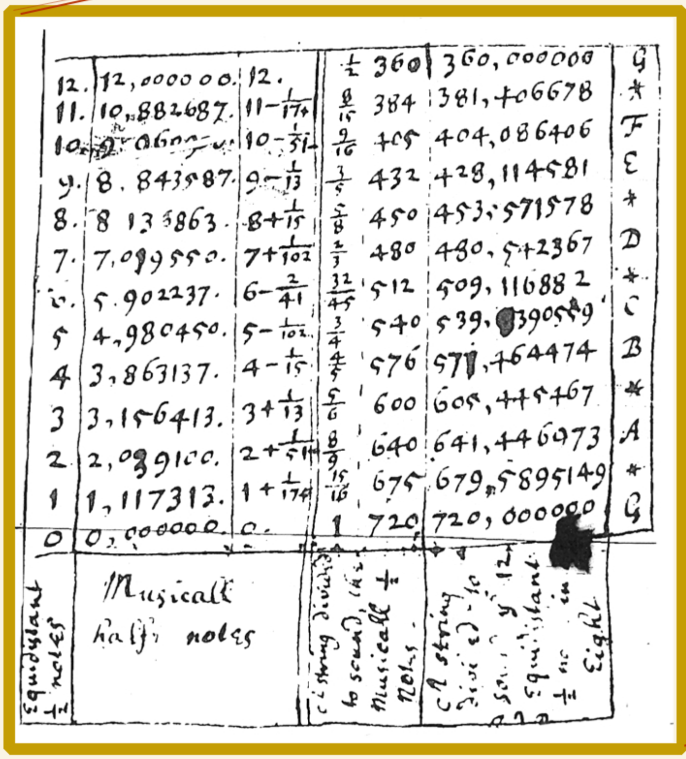


“*in Experiments, where Sense is Judge*”

Isaac Newton’s

TONOMETER and COLORIMETER



Charles R. Adams

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http://www.oughtred.org/josplus.shtml

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Front Cover

Isaac Newton, manuscript, "*Musicall halfe notes*,"
Cambridge University Library, *Add. Ms.* 3958.2 (c. 1665)

Back Cover

Isaac Newton, manuscript, 'Music and Mathematics,'
Cambridge University Library, *Add. Ms.* 4000, fols. 109^{v}-110^{r} (c. 1665)

"in Experiments, where Sense is Judge"

Isaac Newton's
TONOMETER and COLORIMETER

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"Thought is a thread of melody running through the succession of our sensations."
(Charles Sanders Peirce, "How to Make Our Ideas Clear" [1878], *Collected Papers*, 5.395)

At the opening of Book III ("The System of the World") of *Principia mathematica*, Isaac Newton wrote:

> "In the preceding books, I have presented principles of [natural] philosophy that are not, however, philosophical but strictly mathematical—that is, those on which the study of philosophy can be based. These principles are the laws and conditions of motions and of forces. But in order to prevent these principles from seeming sterile, I have illustrated them with some philosophical scholiums, treating topics that are general and that seem to be most fundamental for [natural] philosophy, such as the density and resistance of bodies, spaces void of bodies, and the motion of light and sounds. It still remains for us to exhibit the system of the world from these same principles" (Newton, *Mathematical Principles of Natural Philosophy* III ([1687]1999:793).

By then, Newton had been researching the 'most fundamental topics' of the *motion of light and sounds* for more than two decades, in a century that was rife with curiosity about the "consent and dissent of visibles and audibles" (Francis Bacon [1561-1626]), and often as not with more emphasis on acoustics and audition than on optics and vision: "Sounds can bring more illumination to natural philosophy than any other quality: that is why the science of music should not be neglected even if singing and concerts were to be entirely abolished and forbidden" (Marin Mersenne [1588-1648]). All the major scholars in the 17th century engaged in some form of audial-visual discourse and Newton was no exception.[1] In a brief and youthful essay "Of Musick" he wrote, for example, "…as too suddaine a change from lesse to greater light offends y^e eye by reason y^t y^e spirits rarified by the augmented motion of y^e light too violently stretch y^e optick nerve: soe y^e suddaine passing from grave to acute sounds is not so pleasant as if it were done by degrees" (Newton, Cambridge University Library, *Add. Ms.* 4000, 138^r, c. 1665; also Pesic 2006, 2014). Later (e.g., *Optical Lectures* [1671-2], "An Hypothesis Concerning the Properties of Light" [1675], and most famously in *Opticks* [1704]), Newton came to describe the elongated shape of the prismatic spectrum of light (an "Image of the Sun", a "Pillar of colours") in succinctly musicological terms:

> The intervals (spaces) occupied by the principal "prismatick Colours, red, orange, yellow, green, blue, indigo, violet," … are determined to be "in proportion as the Differences of the Lengths of a Monochord which sound the Tones in an Eight, *sol*, *la*, *fa*, *sol*, *la*, *mi*, *fa*, *sol*," that is, "proportional to … the Numbers, 1, 8/9, 5/6, 3/4, 2/3, 3/5, 9/16, 1/2, which express the Lengths of a Monochord sounding the Notes in an Eighth; … and so to represent the Chords of the Key, and of a Tone, a third Minor, a fourth, a fifth, a sixth Major, a seventh and an eighth above that Key" (Newton, *Opticks* [1704]1979:295, 305, 128).

This matho-musical characterization of the colorific spectrum, a tuning of the rainbow, was grounded not so much on speculations as to the 'motions of nerves'—although that was a prevalent contemporary interest (see Kassler 1984; Rousseau 1989, 1991, 2004; Wallace 2003; Wardhaugh

2008[c])—as on extensive and systematic empirical measurements of the order and distribution of the refracted spectral colors, as they appeared in diligent observations, that Newton had begun about 1666.[2]

Although Newton was one of the principal architects of the modern science of light and colors, including basic phenomena of their perception and cognition, his quantitative methods have long since been forgotten or forsaken for other techniques:

> "The foundation of color science as now accepted was laid 300 years ago by the great mathematician and physical scientist Isaac Newton. … An effective way to summarize approximately the additive relationships between [spectral] colors was devised by Isaac Newton and is known as *Newton's Color Circle*… . [It is] a kind of map showing saturated colors on the circumference, and progressively less saturated colors closer to the center. It is a useful way to visualize color relationships, because it includes information about both hue and saturation. However, it cannot accurately predict the results of additively mixing two colors, since it does not assign numerical values to colors. … Although Newton's color circle suggests qualitatively how to predict the result of additively mixing [spectral] colors, the C. I. E. [color space] diagram provides a way of precisely and quantitatively predicting the result. [Newton's color circle] … was the first successful method that summarized consistently several important relationships between colors (e.g., complementarity, progression of spectral hues, variation in saturation, and additivity of red and blue to yield purple). Yet it failed to provide a *numerical* measure for color and thus lacked accuracy and reproducibility" (Williamson and Cummins 1983:11, 20, 21, 60).

The actual approach Newton used when measuring prismatic colors, which indeed gave the spectral dispersion 'proportional to the numbers' and did predict, quantitatively, the results of mixing spectral colors, had enlisted the techniques of a robust and mathematical *scientia musica*. Ironically, historical awareness of those practices has often been obscured by the persistent (albeit erroneous) assumption that the 'musical' allusions in Newton's optical researches amounted to a little more than unscientific dabbling in numerology, 'a strangely mystical … almost weird obsession':

> "Newton's work touches the science of sound in only two points of contact both of small content, and yet of great significance, since one of these contacts is strangely mystical [that is, the prismatic tuning], and the other very logical [*Principia mathematica*, Book II, Propositions 47–50 relating that 'the velocity of propagation of a pulse in an elastic fluid is directly proportional to the square root of the elasticity and inversely proportional to the square root of the density'], and together they represent the distinct boundary between the old and the new in the science of sound. Newton in his *Opticks* ... when treating the colors of thin transparent bodies, such as soap bubbles, thin plates of mica, films of grease, etc. and particularly of the colors known as 'Newton's Rings,' says: 'The limits of the seven colours, red, orange, yellow, green, blue, indigo and violet in order are to one another as the cube roots of the squares of the eight lengths of a chord which sound the notes of the musical scale' [after *Opticks* {1704}1979:212, 225, 305]. He believed that the seven colors were seven distinct homogeneous unchangeable kinds of light and that the proportional space occupied by each in the spectrum was due to an inherent property of the color. ... The ratios of these tones ... were thought by many philosophers and astronomers to furnish the key to many of the mysteries of nature. Newton's obsession with this almost weird connection between physical facts and metaphysical speculation caused him to miss the discovery of the very important fact of the continuous gradation of color and of the extension of the spectrum beyond the visible rays. Sir J. J. Thomson remarks: 'It was, I think, the siren's song of these harmonies that lured Newton to this false conclusion'." (from the Presidential Address at the Annual Meeting of the Acoustical Society of America in 1932, cited in Miller 1935:24-26).

It is not my purpose here to explore the origins or perpetuations of this (or any other) "seven-color misinterpretation of Newton's theory" of white light and colors (Shapiro 1994:626). Rather, more simply, I will trace the developments and interrelationships of two circular measuring devices which he innovated, an early TONOMETER and a late COLORIMETER—both inspired by the circular slide rules invented by William Oughtred [1575-1660]—and do so largely in Newton's own terms, those of "a mathematician to his toe-tips. From the first, he regarded everything in his view with an outward eye attuned to arithmetical and geometrical niceties, and with an inner vision which sensed the mathematicisable under-structure in all things" (Whiteside 1995:412).[3]

While he was a young student Newton undertook an extensive study in 'musical' analysis that was shaped within the mixed (applied) mathematical discipline of *canonics* or *harmonics*, building on techniques that had been developed even before Euclid wrote a treatise on the subject: κατατομή κανόνος (*Sectio canonis*), *The Division [Cutting] of the Canon* or monochord (Gk. *kanōn*; L. *regula*) [on the ancient procedures see Barker 1992, 2007; Barbera 1991; Creese 2010]. Canonics, rather modernized, was ubiquitously practiced throughout the 17th century and was well known to mathematicians and musicians alike (Kassler 1979; H. F. Cohen 1984; Burnett, Fend and Gouk 1991; Coelho 1992; Erlmann 2010; Moyer 1992; Gouk 1999; Gozza 2000; Vendrix 2008; Wardhaugh 2008[b]; Urreiztieta 2010; Pesic 2014).

> "The science of canonics, which was invented by the Pythagoreans [such as Philolaus of Croton, Archytas of Tarentum, Plato, Euclid, Nicomachus of Gerasa], is in all essentials applied metrical geometry. According to its procedures, each musical pitch is represented by a length on the canon [*kanōn*, tensioned string; **Figure 1**] or rule, a length that is practically measurable. For, given any magnitude on the rule, it is possible to discover what the number is to which that magnitude has a relation.
>
> … inverse proportion [was] the most imaginative harmonic concept in the entire mathematical arsenal of the Pythagoreans. … Since the Pythagoreans could not measure the actual rates of vibration, nor compute the actual amounts of tension involved in the production of [perceived] pitch, they did the next best thing: … basing their computations on what they could measure to a certainty—the lengths of strings … they inverted the numerical proportions that they obtained from these and arrived in this way not at the actual, experienced world of sound, whose causal factors could only be imagined, but a geometric world made real in numbers.
>
> … the frame of reference is the *diastema*—that 'distance' [interval] or continuous extension on the canon, whose relation to another 'distance,' or commensurably continuous extension on the canon, is expressible as a *logos*, or numerical ratio" (F. R. Levin, in Nicomachus of Gerasa, *Manual of Harmonics*, 1994:36 [n. 8], 143, 146).

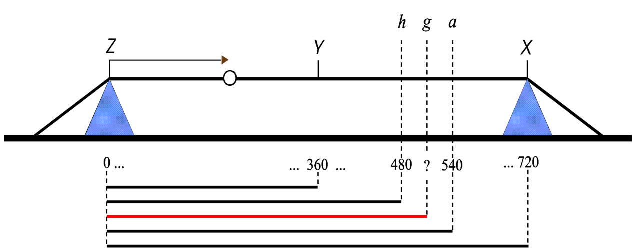


Figure 1: *Kanōn*

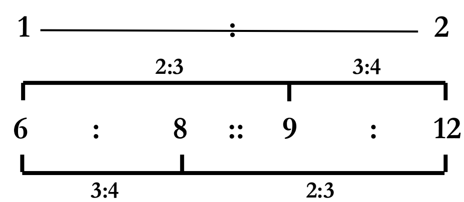


Figure 2: *System, Harmony*

Canonics was an art of measurement (*metritikē technē*) and part of the practical art of calculation (*logistikē*). But the ancient techniques of 'dividing the rule' were largely restricted to 'cutting at the joints,' that is, no further than was possible by the use of integers alone, the relationships of those numbers then revealing well-ordered sets of intervals (integer ratios) often called a *systēma* or *harmonia* (**Figure 2**). The arithmological limitation was consistent with general philosophical and musicological doctrines whereby relationships only amongst sustained discrete sounds of different pitches, indexed to integers, were regarded as having any properly 'musical' significance. Continuously variable sounds were, by definition, *acoustical* but not *musical* material, the two categories being thought mutually exclusive and unrelatable.[4] In the 2nd century CE, Claudius Ptolemy applied the concepts of the continuous (Gk. *sunexis*, L. *continuus*) and the discrete (Gk. *diakrisis*, L. *terminabilis*), in both his works on *Harmonics* and *Optics*, to the characterization of natural phenomena, such as rainbows.[5] He approached the idea that the scales of audible tones and the rainbow order of visible colors exhibited characteristics of organization that were at once continuous and discrete, an observation oft-repeated in subsequent works, for example:

> "Ptolemy appears to side with the Pythagoreans insofar as he also thinks that the basis for high and low pitch does not reside in quality, but rather in quantity [magnitude], because he holds that more compact, thinner bodies emit high pitch, and less compact, very large bodies emit low pitch. ... Pitches not defined by a point of distinction [i.e., those exhibiting a tonal continuum] occur in this manner. Just as when a rainbow is observed, the colors are so close to one another that no definite line separates one color from another—rather it changes from red to yellow, for example, in such a way that continuous mutation into the following color occurs with no clearly defined median falling between them—so also this may often occur in pitches. ... Thus some non-unison pitches are continuous, others discrete. Continuous pitches are such that the difference between them is joined by a continuous line, and the high pitch—or the low—does not maintain a clearly defined position. Discrete pitches, on the other hand, have their own positions, just as unmixed colors do, between which a difference is perceived by virtue of a clearly established position. Non-unison pitches that are continuous are not considered by the faculty of harmonics, for they are dissimilar from each other and yield no single entity of sound. Discrete pitches ... are subject to the harmonic discipline, for the difference between dissimilar pitches separated by an interval can be comprehended" (Boethius, *Fundamentals of Music* 5.4-6, [6th century CE]1989:166-167).

Figure 1 shows partial lengths of a whole tensioned string *ZX*, in numbers—**360**, **450**, **540**, and **720**—used by Newton. **Figure 2** displays the canonical *system* of these lengths/numbers, reduced by a factor of **60** to the 'musical proportion' (Gk. *mousikē analogia*) **6:8::9:12**, in least ratios revealing the mutually reciprocal rationality of the medial terms, the *harmonic* (***h*=8**) and *arithmetic* (***a*=9**) means relative to the extreme terms *YX* (=**1:2**). The interval ratios (magnitudes) delimited by these mean terms are unequal, while the (unexpressed) *geometric* mean (***g***) is that point which occurs in equal ratios to the extremes, ***Y:g::g:X***. The totality of relationships within the continuous extension of the geometrical line (= tensioned string) constitutes an exact metrical geometry, even while musicological theorists continued to discuss their subject primarily in a discretized language of arithmetical terms.[6]

Johannes Kepler [1571-1630] was especially intrigued with the *geometrical* characteristics of a *system* of sounds, a tuning, in his endeavors to explore the *systematic motions* of celestial physics. He geometrized the problems of canonics and harmonics in a very literal and direct way, making arithmological descriptions of musical tones and intervals derivative and secondary:

> "[Now] let us think of the [tensioned and resounding] string in all respects as if it is not a straight line but a circle" (Kepler, *Mysterium cosmographicum*, 1596[1621]1981:131).

> "We must seek the causes of the harmonic proportions in the divisions of a circle into equal aliquot parts, which are made geometrically and knowably, that is, from the constructible regular plane figures. … [Moreover,] as far as music is concerned, it is sufficient that a string stretched out straight can be divided in the same way as when it is bent round into a circle it can be divided by the side of the inscribed figure [i.e., a regular polygon]" (Kepler, *The Harmony of the World* [1619]1997:9, 144).

Analyzing the relationships between the parts and whole of a continuous circle that was 'severed' (partitioned or divided) by inscribed regular polygons, Kepler derived a 'complete system' of musical tuning. He expressed it in terms of partial string-lengths and an orderly succession of intervals 'numbered' within the compass of the "two signal termini of a measuring rod," a *kanōn*, comprising the rational magnitude of an 'octave' **360:720**, the interval ratio **1:2** [as in *YX*, **Figure 1**] (Kepler, *Epitome of Copernican Astronomy* IV.4 [1618-1621]).

> "Thus, … by a multiplex form of division into aliquot parts … there occur in one octave overall thirteen strings, in the … lowest terms. I have inserted among them all the smallest intervals according to the natural order in a full and perfect functional system" (Kepler, *The Harmony of the World* III.8 [1619]1997:195).

Kepler's geometrical sectioning of an encircled stretched string explored the possibilities of extending equal-ratio divisions of a continuum from the first mean proportional (that is, the geometric mean g, accomplished by a digon or diagonal) to subsequent mean proportionals. That was more easily accomplished with a geometrical technique than a corresponding algebraic one.[7] The first mean proportional of a rational magnitude XY is $\sqrt{XY}$ or $(XY)^{1/2}$, followed by $(XY)^{1/3}$ [trigon], $(XY)^{1/4}$ [tetragon], $(XY)^{1/5}$ [pentagon], $(XY)^{1/6}$ [hexagon], $(XY)^{1/7}$ [heptagon], and $(XY)^{1/8}$ [octagon], etc. The vertices of these polygons mark off integral numbers of equal arc lengths comprising the whole circumference of the circle, and therefore they also imply 'equal-ratio' divisions of a length of tensioned string.[8]

Rethinking the mathematics of canonical division, Kepler had attained a much improved way not only for reexamining the relationships of discrete and continuous phenomena but also for integrating audial-acoustical questions with visual-optical astronomy. And he was delighted when he learned about the kinetic geometrical form of the "wonderful business of logarithms (of the illustrious lord, Baron Napier) as in those, which arise in the circle, there is a sort of calculating machine of all the multiplications and divisions which can ever take place" (Kepler, *The Harmony of the World* IV.7 [1619]1997:373).

> "A logarithmic table is a small table by the use of which we obtain a knowledge of all geometrical divisions and motions in space; by a very easy calculation it is picked out from numbers progressing in continuous proportions" (John Napier, *The Description* … and *The Construction … of the Wonderful Canon of Logarithms* (1614, 1619).

Isaac Newton's mathematical interests also prioritized geometrical thinking. His major project in optics continued the researches of the mathematician and classical scholar Isaac Barrow [1630-1677], his mentor and predecessor in the Lucasian chair at Cambridge. And Barrow had once lectured on '*Acoustics*' as well as 'that Part of Mathematics termed *Music*, *Harmonics* or *Canonics*.'

> "… if the Figure of Air's Undulation could be the same way discovered, by which Sound is performed, and the Sense of Hearing impelled, there would doubtless arise thence a new Part of Mathematics to be celebrated by the Name of *Acoustics*, or the Science of Sounds. … However, as to what pertains to the Sense of Hearing, because of the certain Division of a sonorous String, or from the Length of two Strings differing according to a certain Proportion, both stretched alike, if both be struck with a Quill, there is found a determinate Habitude of the Sounds produced, which wonderfully answers to the Proportion of the Strings according to the Degrees of Sharpness or Flatness, and consequently is explicable by Numbers (or at least by Right Lines), hence arises that Part of Mathematics termed *Music*, *Harmonics* or *Canonics*, which (as some love to speak) is subaltern to Arithmetic; but perhaps it would be more rightly subordinate to Geometry" (Barrow 1970:17-18, 24-25).

"Barrow—himself apparently self-taught—seems, in his public lectures [*Mathematical, Geometrical, Optical*] at Cambridge and London from the 1650's, to have been the first university teacher in England systematically to explore the riches of the Greek mathematical opus" (Whiteside 1961:272). He published editions of the works of Euclid, Archimedes, Apollonius, and Theodosius of Bithynia, and he was also a Regius Professor of Greek, Gresham Professor of Geometry, and later Master of Trinity College. For Barrow, proper mathematical objects in all cases were the same—quantity, that is, continuous magnitude—insofar as "there exists in fact no other quantity different from that which is called magnitude, and further, it alone is rightly to be counted the object of mathematics, which investigates and demonstrates first of all its general properties, then its nearest forms and the affections congruent to them" (Barrow, *Mathematical Lectures* 39-40). Thus, he considered the various sciences of nature as "rather so many examples only of geometry, than so many distinct sciences separate from it." The alignment of 'mathematics' with problems of continuous magnitude and their measurements was a modern scientific development (Koyré 1968; Roche 1998; Neal 2002). It "contained and circumscribed all of mathematics within the bounds of geometry," Barrow emphasized, as Kepler and Galileo and others had before, so that mathematical numbers were nothing other than the convenient signs of geometrical magnitudes.

> "Citing the authority of Plato's *Philebus*, Barrow distinguished between 'mathematical' and 'transcendental or metaphysical' numbers. The latter, which are used to count collections of items of generally like sort, are in a sense premathematical. They belong to a common discourse, which follows no firm rules on which to base a mathematical science. Mathematical numbers, by contrast, presuppose a uniform basis of measure. They enumerate collections of units precisely equal to one another or they measure magnitudes with reference to a common unit. But numbers are not abstract quantities in themselves. Their existence depends on the units underlying them, because that is where the criteria of quantity and the means of measure reside" (Mahoney 1990:185-186).[9]

At the outset of Newton's optical investigations geometry was paramount and the primary objects of interest were the measurements of the continuous magnitudes inherent in the natural phenomena of, specifically, 'the reflections, refractions, inflections and colors of light.'

> "… the generation of colors includes so much geometry, and the understanding of colors is supported by so much evidence, that for their sake I can attempt to extend the bounds of mathematics somewhat, just as astronomy, geography, navigation, optics, and mechanics are truly considered [mixed] mathematical sciences, even if they deal with physical things: the heavens, earth, seas, light, and local motion. Thus although colors may belong to physics [that is, 'nature'], the science of them must nevertheless be considered mathematical, insofar as they are treated by mathematical reasoning. … I therefore urge geometers to investigate nature more rigorously, and those devoted to natural science to learn geometry first. Hence the former shall not entirely spend their time in speculations of no value to human life, not shall the latter, while working assiduously with absurd methods perpetually fail to reach their goal. But truly with the help of philosophical geometers and geometrical philosophers, instead of the conjectures and probabilities that are being blazoned about everywhere, we shall finally achieve a natural science supported by the greatest evidence" (Newton, *Optical Papers*, *Optica* II, *Lecture* 1, c. 1671, A. Shapiro (trans.) 1984:439).

Newton's interest in a natural-philosophical *geometrikē technē*, including the musical metrical geometries of canonics, was quite general and extremely far-reaching. He was heir of the diverse traditions of Greek geometry (see, for example, Whiteside 1961; Grattan-Guinness 1996), even while advancing in novel and modern directions. Later in life he sought out, in the older philosophical literature, 'adumbrations' of his own singular accomplishments, becoming a novel historian by enlisting his own disciplinary innovations, mastering and mustering, for one, the vast technological resources of mathematical astronomy to reconstruct the very 'origins of human civilization' (Buchwald and Feingold 2013).

It has often been stressed that "… the philosophy of mathematics endorsed by Newton can be termed physicalist insofar as it implies a belief that mathematical representations should be closely aligned with the properties of physical bodies and their motions" (Guicciardini 2009:313; see also Whiteside 1961). This topic is far too vast to review here, but some quotations can convey the general tenor of Newton's thinking on 'geometry coextensive with physics.'

> "[G]eometry neither teaches how to describe a plane nor postulates its description, though this is its whole foundation. To be sure, the planes of fields are not formed by the practitioner [*ab artifice*] but merely measured. Geometry does not teach how to describe a straight line and a circle but postulates them; in other words, it postulates that the practitioner has learnt these operations before he has attained the threshold of geometry. … Both the genesis of the subject matter of geometry, therefore, and the fabrication of its postulates pertain to mechanics. Any plane figure executed by God, Nature or any Technician [*a Deo Natura Artifice quovis confectus*] you are to measure by geometry by the hypothesis that they are exactly constructed. …

… [a] technician is required and postulated to have learnt how to describe straight lines and circles before he may begin to be a geometer {'a world measurer'}. And it consequently does not matter how ~~by what mechanical means~~ they shall be described [*Ideo refert quo modo* ~~qua ratione mechanica~~ describantu]. Geometry does not posit modes of description: we are free to describe them [geometrical figures] by moving rulers around, using optical rays, taught threads, compasses, the angle given in a circumference, points separately ascertained, the unfettered motion of a careful hand, or finally any mechanical means whatever. Geometry makes the unique demand that they [the figures] are described exactly [*Id solum postulat Geometria ut describantur exacte*]." (Newton, *Geometria*, *Mathematical Papers* VII:248-561).

"… the whole subject of *mechanics* is distinguished from *geometry* by the attribution of exactness to *geometry* and of anything less than exactness to *mechanics*. … For the description of straight lines and circles, which is the foundation of *geometry*, appertains to *mechanics*. … *geometry* is founded upon mechanical practice and is nothing other than that part of *universal mechanics* which reduces the art of measuring to exact propositions and demonstrations" (Newton, "The Author's Preface to the Reader," *Principia mathematica* [1687]1999:382-3).

For Newton, mathematically interesting entities (continuous geometric magnitudes) were those regularly 'generated' by the *motions* or *actions* of some 'artifex.' Moreover, toward the improvement of mathematical reasoning, it was unnecessary to decide whether that 'practitioner' be god, a technician, a natural process, an artisan (or a musician plucking stretched strings). Any motions generate geometrical figures that can be described exactly. General concepts of kinematic geometry were inherent to the infrastructure of 'the calculus.'

"Mathematical quantities I consider here not as consisting of indivisibles, either parts least possible or infinitely small ones, but as described by a continuous motion. Lines are described and by describing generated not through the apposition of parts but through the continuous motion of points; surface areas are through the motion of lines, solids through the motion of surface-areas, angles through the rotation of sides, times through continuous flux, and the like in other cases. These geneses take place in the physical world and are daily enacted in the motion of bodies visibly before our eyes. And in much this manner the ancients, by 'drawing' mobile straight lines into stationary lines, taught the genesis of rectangles.

By considering, then, that quantities increasing and begotten by increase in equal times come to be greater or lesser in accord with the greater or less speed with which they grow and are generated, I was brought to seek a method of determining quantities out of the speeds of motion or increment by which they are generated, and, naming these speeds of motion or increment 'fluxions' of the quantities generated, and the quantities so born 'fluents', I fell in the years 1665 & 1666 upon the method of fluxions which I have formerly imparted to friends and which I have here employed in the quadrature of curves" (Newton, "The Method of Fluxions", *Mathematical Papers* VIII:107; see also Newton, *Two Treatises on the Quadrature of Curves, and Analysis by Equations of an Infinite Number of Terms*, trans. John Stewart (London 1745).[10]

"In general summary of the later 17th century attitude to analytical techniques in geometry … it seems true to say that Cartesian analysis, while accepted as a useful form of proof, was looked upon as essentially eliminable by the substitution of an exactly corresponding synthetic form. Newton's appendix to his *Arithmetica universalis*—an eternal worry to those historians who have tried to read 19th century attitudes into 17th century mathematics—essentially summarizes a prevailing attitude:

"Equations are expressions of arithmetical computation and properly have no place in geometry except in so far as truly geometrical quantities (that is, lines, surfaces, solids, and proportions) are thereby shown equal, some to others. Multiplications, divisions and computations of that kind have been recently introduced into geometry, unadvisedly and against the first principle of this science. {For anyone who examines the construction of problems by the straight line and circle devised by the first geometers will readily perceive that geometry was contrived as a means of escaping the tediousness of calculation by the ready drawing of lines.} Therefore these two sciences [arithmetical computation and geometry] ought not to be confounded. {The Ancients so assiduously distinguished them one from the other that they never introduced arithmetical terms into geometry}; while recent generations by confounding them have lost that simplicity in which all geometrical elegance consist" (Newton, [*Mathematical Papers* V: 429], quoted in Whiteside 1961:341; see also Grattan-Guiness 1996; Pycior 1997; Guicciardini 2009: 64).

> "Their [the ancients'] method is much more elegant than that of Descartes. For he attains the result by means of an algebraic calculus which, if one transcribed it in words (in accordance with the practice of the Ancients in their writings), is revealed to be boring and complicated to the point of provoking nausea, and not to be understood. But they attained it by certain simple proportions, for they judged that what is written in a different style is not worthy to be read, and consequently they concealed the analysis by which they had found their constructions" (Newton, *Mathematical Papers* IV:277).[11]

> "In later life Newton came to object strongly to the Cartesian approach, calling it the 'Analysis of the Bunglers in Mathematicks,' … 'for these computations [the arithmetic of variables] … often express in an intolerably roundabout way quantities which in geometry are designated by the drawing of a single line' " (Newton, *Geometrica curvilinear*, c. 1680, [*Mathematical Papers* IV:421], quoted in Gjertsen 1986:170-171).

Newton's research into canonics was of a piece, however minor, with his generally 'geometrical' orientation. The ratios and proportions of a system of musical tones, as all others, were geometrical quantities (although we should not confuse the simple static geometry of the straight line with the dynamic geometries of orbital mechanics or ballistic trajectories). But more significantly, ideas from the field of musical-metrical geometry (the 'cutting of the *kanōn*') and the divisions of a tensioned string directly informed Newton's inquiries into the phenomena of prismatic refraction. A prism was said to 'refract' (sever, break, cut, divide, partition) the "whole mass" of the white light of the sun and to distribute its colorific constituents into an orderly linear succession, creating the appearance of the *spectrum*, an "Image of the Sun". Consequently, the intervals (spaces) occupied by the principal "prismatick Colours, red, orange, yellow, green, blue, indigo, violet," … in that distribution, were appropriately defined "in proportion as the Differences of the Lengths of a Monochord [*kanōn*]". Newton mastered the principles of canonics in a series of youthful exercises that he called "a discourse of y[e] motion of strings sounding … & of y[e] Logarithms of these strings, or distances of y[r] notes" (CUL, *Add. Ms.* 4000, fol. 137[v]). The "discourse", which survives in some nineteen manuscript pages, was primarily mathematical, in contrast to a short essay "Of Musick" where the emphasis was on musicological and affective criteria such as the 'harshness' or 'pleasantness' of tones and modes and intervals.[12]

Newton's TONOMETER
"a discourse … of strings sounding"

In meticulously detailed student notebooks (c. 1664-5), Newton analyzed a system of musical tuning that was very nearly identical to Kepler's, albeit more exhaustively and elegantly symmetrical. He calculated and tabulated "How y[e] string 1 or 720 is to bee divided y[t] it may sound all y[e] musicall notes & [musicall] halfe notes in an eight" (Newton, CUL, *Add. Ms.* 4000, fol. 105[v]) and thereby described a canonical system which is summarized in **TABLE 1**.[13]

The whole reference magnitude of Newton's system ("the string 1 or 720") contains an orderly succession of partial magnitudes. By ratios, string-lengths progressively *decrease* from whole (**1/1**) to half (**1/2**), inversely mirroring those which *increase* from half to whole, the greater magnitudes being *inclusive* of the lesser.[14] String-lengths are proportional to the resounding frequencies of their fundamental vibrations (perceived pitches). So 'by the numbers,' string-lengths *descending* from **720** to **360** are inversely proportional to the fundamental pitches they resound *ascending* in an orderly melodic succession (or scale). The tuning system is completely symmetrical in the relations of means to extremes, that is, to the *geometric mean* (g) such that $720:g :: g:360$ (the [first] mean proportional g, $(720 \times 360)^{1/2}$, is located at **509.1168825** in this distribution of string-lengths/pitches). Pairs of intermediate 'extreme' terms, such **640** *and* **405** or **540** *and* **480** (the harmonic and arithmetic means), are located at equal-ratio distances from the greatest extremes as well as from the geometric mean. The mean-extreme symmetry of the tuning is rational in both its ‘chromatic’ (smaller interval ratios) and ‘diatonic’ (larger interval ratios) patterns, that is, in respect to expressing the *successive* intervals between the string-lengths in terms of some few small ratios.[15]

TABLE 1
Synopsis of Isaac Newton’s Canonical Tuning System and Symmetries

	G	**F#**	**F**	**E**	**E♭**	**D**	**C#**	**D♭**	**C**	**B**	**B♭**	**A**	**A♭**	**G**
	G	**A♭**	**A**	**B♭**	**B**	**C**	**D♭**	**C#**	**D**	**E♭**	**E**	**F**	**F#**	**G**
decrease	**1/1**	**15/16**	**8/9**	**5/6**	**4/5**	**3/4**	**32/45**	**45/64**	**2/3**	**5/8**	**3/5**	**9/16**	**8/15**	**1/2**
increase	**1/2**	**8/15**	**9/16**	**3/5**	**5/8**	**2/3**	**45/64**	**32/45**	**3/4**	**4/5**	**5/6**	**8/9**	**15/16**	**1/1**
lengths	**720**	**675**	**640**	**600**	**576**	**540**	**512**	**506¼**	**480**	**450**	**432**	**405**	**384**	**360**
pitches	**360**	**384**	**405**	**432**	**450**	**480**	**506¼**	**512**	**540**	**576**	**600**	**640**	**675**	**720**

‘chromatic’ ratios	16/15	135/128	16/15	25/24	16/15	135/128	2048/2025	135/128	16/15	25/24	16/15	135/128	16/15

‘diatonic’ ratios	9/8	16/15	10/9	9/8	10/9	16/15	9/8
	T	**S**	**t**	**T**	**t**	**S**	**T**

The interval ratios of Newton’s tuning system, as specified ‘musically,’ are few and well-organized but unequal in size. He also examined some dozen equal-ratio (‘geometrical’) divisions, including “a string ([1 or] 720) divided into 12 (geometrically progressionall) parts, yt it may sound ye 12 exact [or ‘equidistant’] ½ notes in an eight” as well as … “ye logarithms of a cord divided into 12 geometricall partes)”, while also tabulating “the proportion wch the musicall notes & halfe notes bear ye one to ye other (viz ye logarithms of ye string sounding them)” (Newton, CUL *Add. Ms.* 4000, fol. 105v; also Newton, CUL *Add. Ms.* 3958.2, fol. ‘scrap page,’ see *front cover*). These divisions or sectionings took the rational form of $1 : (\sqrt[x]{2})^n$ or $1 : 2^{n/x}$ for equal-x divisions of the “string 1 or 720”, in cases where $x = 12, 20, 24, 25, 29, 36, 41, 51, 53, 59, 60, 100, 120, 612$. Determinations of the ‘roots and powers’ were facilitated by logarithmic calculations and tables, for which Newton most frequently used the common or ‘Briggs’ form with decimal values, where $\log 1 = 0$, $\log 10 = 1$, $\log 100 = 2$, etc.[16]

Consistent with the historical legacy of a canonical metrical geometry, Newton conceived the tuning system in terms of the rational sectioning or partitioning of a *unitary continuous magnitude,* a “1 or 720”, writing in “Of Musick” (c. 1665), “… some one sound [‘ye Cliffe or Key of ye song’, … or ye Ground sound’] must be pitched upon to wch all ye musick must be more especially referred yn to any other sound (as number is to an unit).” But his methods, like those of Isaac Barrow and other contemporary mathematicians, invoked more modern concepts of ‘number’ and ‘unit’, than the ‘ancient’ understandings allowed (also see Pesic 2010, 2014).

> “For the ancients it [unit] was (as a ‘pure’ unit) the ‘principle of number’ and as such, simply indivisible…; for the ‘moderns’ it is, ‘*as something continuous*’ (ut quid continuum), divisible into as many (equal) parts as you please: ‘*When arithmetic wishes to imitate in some way the infinite divisibility of geometry, it supposes a unit*

or a one which is something whole, as it were, but divisible into as many parts as you please.' … according to Wallis, a 'ratio' [Gk. *logos*], a 'relation' underlies every 'number'. In his discussion of the fifth book of Euclid's *Elements*, … he says, … *'the whole of arithmetic itself seems, … to be nothing but a theory of ratios, and the numbers themselves nothing but the indices of all the possible ratios whose common consequent is 1, the unit. For when 1 or the unit is taken as the [unique] reference quantum, all the rest of the numbers (be they whole, or broken or even irrational are the 'indices' or 'exponents' of all the different ratios possible in relation to the reference quantum'* " (Klein 1968:218-220, citing John Wallis [1616-1703], *Mathesis universalis*, 1657).

With a 'reference quantum' being 'something continuous' and consisting of magnitudes expressed as successions of ratios, these new mathematical ideas were most felicitous for modern canonics, partitioning a whole stretched string. Moreover, they resonated nicely with the core notion of a *logarithm*, that is, in John Napier's coinage, 'a number of a ratio' (*logos* + *arithmos*), insofar as that made possible for mathematical numbers to be 'signs of magnitudes' expressed as ratios (Barrow). In the simplest sense a progression of numbers, such as **1, 2, 4, 8, 16, 32…**, is rewritten as an 'arithmetic' succession of the exponents of one 'equal ratio,' $(2:1)^0$, $(2:1)^1$, $(2:1)^2$, $(2:1)^3$, $(2:1)^4$, $(2:1)^5$…, 'multiplying the one magnitude'. Reciprocally, 'dividing the one [or whole] magnitude' as a 'harmonic' succession of the exponents of an 'equal-ratio' gives $(1:2)^0$, $(1:2)^1$, $(1:2)^2$, $(1:2)^3$, $(1:2)^4$, $(1:2)^5$… for the reciprocal progression **1/1, 1/2, 1/4, 1/8, 1/16, 1/32…** . Any magnitude (ratio) is measurable as multiples or sub-multiples of other magnitudes (ratios), an efficient relationship between measures and the measured.

Much of Newton's "discourse … of strings sounding" consisted of detailed tabulations of the interrelationships between all the tones and intervals ('chromatically' and 'diatonically') in his tuning system (**TABLE 1**), according to different canonical measurements. So, for example, "a string ([1 or] 720) divided into 12 (geometrically proportionall) parts, y[t] it may sound y[e] 12 exact ½ notes in an eight" ["To sound the 12 equidistant ½ notes in an eight"] (Newton, CUL *Add. Ms.* 3958.2, fol. 30[v]) was compared (**TABLE 2**) to "How y[e] string 1 or 720 is to bee divided y[t] it may sound all y[e] musicall notes & [musicall] halfe notes in an eight" (Newton, CUL *Add. Ms.* 4000, fol. 105[v]).

TABLE 2
Diatonic String-Lengths
(according to Newton's 'Musical' and 'Geometrical' Divisions)

1/1	**8/9**	**5/6**	**3/4**	**2/3**	**3/5**	**9/16**	**1/2**
720	**640**	**600**	**540**	**480**	**432**	**405**	**360**
$1:(^{12}\sqrt{2})^0$	$1:(^{12}\sqrt{2})^2$	$1:(^{12}\sqrt{2})^3$	$1:(^{12}\sqrt{2})^5$	$1:(^{12}\sqrt{2})^7$	$1:(^{12}\sqrt{2})^9$	$1:(^{12}\sqrt{2})^{10}$	$1:(^{12}\sqrt{2})^{12}$
720.000	**641.447**	**605.445**	**539.391**	**480.542**	**428.115**	**404.086**	**360.000**

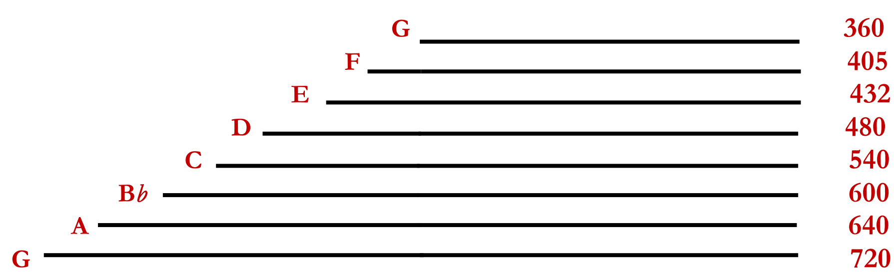
Between these determinations (signposts along the continuum) there are obviously as many intermediate magnitudes as one might care to identify, the continuous magnitude of the whole stretched string being subject, by a variety of techniques, to infinite divisibility. The notation here facilitates recognition of additional mean proportionals, such as $1:(^{12}\sqrt{2})^6$, but also any further parts thereof, such as $1:(^{12}\sqrt{2})^{6.312826}$. Newton often used six place decimal expressions, that is, division up to the millionth (10^{-6}) part of an integer or any 'unit' (as in dividing the "1 or 720"). Conversely, any of Newton's 'equidistant' calibrations could be used to measure the magnitudes of those ('musical') intervals (or any other ratios) with considerable accuracy. He wrote out many of these 'logarithmically,' providing a 'number of a ratio.' In one instance a table measuring some 132 musical intervals in 36 equal magnitude groups "…shews y^e distance of any two notes. As y^e distance of C & E is [G—]B, or a 3^d, or 3,863137 halfe notes. Of B & E tis a 4^{th}, or 4,98045 halfe notes. Of B & F tis 6,097763 halfe notes, or [a] greater 4^{th} [or] a 5^t*b*, by 0,095526 halfe notes &c." (Newton, CUL, *Add. Ms.* 4000, fol. 104^v and fol. 106^r). For example, for the interval ratios **64:45**, **36:25** and **40:27** he listed, respectively, the values **6.097763**, **6.312826**, and **6.804487** relative to an equal-12 division of the whole string-length "1 or 720". At other times, when listing 'equidistant' parts, he simply used the approximate integer 'exponents' of the different ratios possible in relation to the reference quantum. Noticing above (**TABLE 2**) that the ('musically' specified) string-length ratio **720:640** (proportionally **9:8**) is a bit greater (larger in size) than the 'geometrically' calculated ratio **720.000:641.447**, *how much* the two ratios differ is a measurable quantity. On an equidistant scale of **12**, calculate $(12/\log 2)(\log 9/8) = 2.0391$; and so, remembering "1 *or* 720", a length very close to **640** (relative to **720**) can be determined: $720:(\sqrt[12]{2})^{2.0391} = 640.0000006$, while the approximate value is $720:(\sqrt[12]{2})^{2} = 641.44\ldots$. Overall, Newton's "discourse … of strings sounding" explicitly identified fifteen different string-lengths (including those indicated by the ratios **64:45** and **45:32** in **TABLE 1**) located within the continuum between the lengths **480** and **540**. The mathematical techniques in play were capable of determining infinitely many more possibilities.

Throughout his youthful investigation into the metrical geometry of a tensioned string, Newton was acquiring the knowledge and skills of mathematical manipulations, much more so than learning about musical or musicological practices as such. He seems to have been especially intrigued with the variety of different ways some given magnitude—here the reference quantum of an 'octave' **720:360** ('canonically' the ratio of whole to half a tensioned string)—could be mathematically described along with evaluating the approximate relationships between the different possible descriptions. Among the techniques used were ratios, fractions, logarithms, solmization syllables, geometrical diagrams, letters, numbers, and trigonometric functions. **TABLE 3** gives a sample of the multi-scalar tabulations of the *exponents* (***n***) in a few equal-***x*** divisions of Newton's *kanōn*.

TABLE 3

Tabulated Exponents (*n*) in Equal-*x* Divisions of an 'Octave' $(2:1)^{n/x}$

(selected diatonic progressions; after Newton, CUL, *Add. Ms.* 4000, fols. 108^r-109^v)



612	508	451	358	254	161	104	0	*equal-612*	**string-lengths ↑**
100	82	72	59	41	27	18	0	*equal-100*	
41	34	30	24	17	11	7	0	*equal-41*	
29	24	21	17	12	8	5	0	*equal-29*	
12	10	9	7	5	3	2	0	*equal-12*	
1/1	8/9	5/6	3/4	2/3	3/5	9/16	1/2	**'musical' division**	

For example, in reference to the interval G—C, string-length ratio **720:540** (∝ **4:3**), the partial length C measures 24 parts of the whole length G (as divided into 41 equal parts), or 59 parts of G (as divided into 100 equal parts), and so for the rest.

‘Circles of Proportion’: Newton’s Circular TONOMETER

Among Newton’s more innovative applications of logarithmic thinking was a (somewhat sketchy) design of a specially calibrated circular TONOMETER (**Figure 3**), a paper instrument for measuring the ‘distances’ (intervals) of ‘musical’ notes by logarithmic scales. The inspiration for this device derived rather directly from William Oughtred’s logarithmically calibrated circular slide rules (*Circles of Proportion and the Horizontal Instrument*, 1632) (**Figure 4**) which Newton had been studying and using at least a year before his canonical investigations (c. 1665).[17]

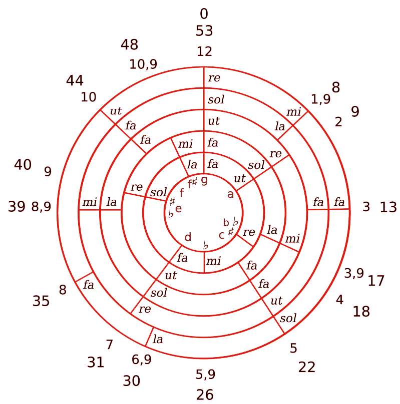


Figure 3
Isaac Newton’s Circular TONOMETER
(after Newton, CUL, *Add. Ms.* 4000, fol. 109r)

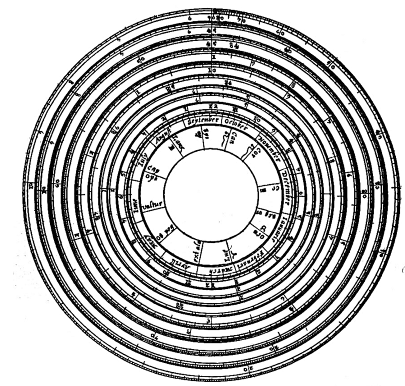


Figure 4
William Oughtred’s *Circles of Proportion*
(courtesy of Chris Sangwin, 2003)[18]

Not only did multiscalar rulers (sliding or no, linear or no), from their inception, facilitate complex calculations, they provided directly proportioned coordination, comparison, and conversions of different scaling systems within a common ‘space’ and defined limits. They made it easy to measure (even casually) one scale against another, much as one could compare heat gradients in ‘degrees’ Celsius, Fahrenheit or Kelvin between the boiling and freezing ‘points’ of water. Circular rules, such as Oughtred’s *Circles of Proportion*, were especially advantageous for coping with cyclic and periodic phenomena (and notably his instrument included an annular ring of the months): calendars, clocks, logarithms, the orbital periods of planets, the trigonometric functions of sine and cosine, and musical tunings (thinking with Kepler of the resounding string as a circle).[19] It was usual for musical theorists to concern themselves with but one cyclic model ‘octave’ (Gk. *diapason*, ratio **2:1**); any series of ‘halvings’ ($1/2^n$) or ‘doubling’s ($2^n/1$) of the tones and intervals contained therein were regarded as perceptual ‘identities’ or ‘affinities,’ as if they were one sound/pitch. In Newton’s tuning system a shorter string and a longer string resounding in the ratio **1:2** are assigned the same letter name, and he sometimes indicated for a tone, say **A**, halving and halving it again as ^{2}**A**, ^{3}**A**, and doubling and doubling it again as **A**^{2}, **A**^{3}, expressing the ‘identities’ in a sort of ‘exponential’ letter notation (Newton, CUL, *Add. Ms.* 4000, fol. 105r).

Newton’s circular TONOMETER (**Figure 3**) displayed a (rudely indicated) ‘chromatic’ scale (in one ‘octave’ ***g*** to ***g*** ∝ **2:1**) in the center. Five concentric circles show a ‘diatonic’ progression of tones, *ut re mi fa sol la fa* [*ut*], beginning on **f c g d** and **a**, in the outer- to inner-most circles.[20]

From various of his manuscripts the succession of intermediate intervals for these tones is known: 9:8 (+) 10:9 (+) 16:15 (+) 9:8 (+) 10:9 (+) 16:15 (+) 9:8.[21] There are two outer rings of numbers, progressing 0 to 12 and 0 to 53. These are series of *exponents* in two equidistant ("geometrically progressionall") scales that measure the magnitudes of the given intervals in *approximate* values. For example, the size of the interval *ut-fa* in all of the progressions is the ratio 4:3. In reference to the middle circle of the TONOMETER, that interval reads 0 to 5 and 0 to 22 on the outer rings, which index the respective ratios $1:(\sqrt[12]{2})^5$ and $1:(\sqrt[53]{2})^{22}$. Their meaning is that the magnitude of that ratio is *approximately* 5 parts of an equal-12 division or *approximately* 22 parts of an equal-53 division of an 'octave' ($1:2^{5/12}$ or $1:2^{22/53}$). That same interval in the second outermost circle (C-*ut*) measures the same, between 5 and 10 on the one scale (with a difference of 5) and between 22 and 44 (with a difference of 22) on the other. More exactly, by the two scales, the interval *ut-fa* (ratio 4:3) measures **$(12/\log 2)(\log 4 - \log 3) = 4.980449991$ (≈ 5)** and **$(53/\log 2)(\log 4 - \log 3) = 21.99698746$ (≈ 22),** which values can be arrived at in logarithmic tables or use of a circular rule by interpolation.[22]

The circular TONOMETER, logarithmically calibrated, was a special application of slide rules. Many such mathematical instruments had been designed during the 17th century for practical purposes in the mixed mathematical sciences, for dialing, gauging, navigation, astronomy, resolving affected equations, as well as for the design and tuning of musical instruments. Newton's prowess with these mechanical means, including his own inventions, for complex calculations was well-known amongst his close acquaintances: "Mr. Newton by helpe of the Logmes graduated on scales that are to lye parallel, at equal distances, or by the helpe of Concentrick Circles so graduated, finds the rootes of æquations; 3 Rulers serve for Cubicks 4 for Biquadraticks &c…" (Letter from John Collins to Henry Oldenberg to Gottfried Leibniz, 1675; see also Newton, *Mathematical Papers* II, 20 August 1672; Cajori 1994:21-23; Sangwin 2002).

Sixty years after Newton limned his tonometric diagram, one Ambrose Warren, who "may have been known to Sir Isaac Newton," published *A Monochord, Called a Tonometer: Explaining & Demonstrating, by an easie Method, in Numbers & Proportion, all the 32 Distinct and Different Notes … of the Gamut or Common Scale of Musick. With their Exact Difference and Distance … Never Before Published* (London, 1725). Warren's TONOMETER consisted of a tensioned string with a length of 2 feet 9¼ inches, scaled into 720 parts, "taking each note of the Thirteen inclusive, in an *Octave* of the common *Scale* or *Gamut*," correlated with an octave divided logarithmically into 32-equal (that is, "geometrically progressionall") intervals, and it was accompanied with large compasses (mechanical dividers) and a circular slide rule to aid the professional tuners of organs, harpsichords and spinets (Kassler 1979:1045-1048). The division of the cyclic octave into 32-equal parts mimics the geometry of a 'compass rose' or 'wind rose' with successive bisections of the four cardinal wind directions (N S W E) [2^2, 2^3 ('major-winds'), 2^4 ('half-winds), up to 'quarter-winds' $2^5 = 32$].[23] Newton's techniques of circular diagramming were also continued by the Cambridge mathematician Robert Smith [1687-1768] (*A Compleat System of Opticks in Four Books, viz. a Popular, a Mathematical, a Mechanical, and a Philosophical Treatise* (1738), and in *Harmonics, or, The Philosophy of Musical Sounds* (1749), as well as in the works of Thomas Young [1773-1829] (Young 2002; and see Kassler 1979; Pesic 2014).

Musical tonometry, applied systematically and world-wide, revolutionized the disciplines of comparative- and ethno-musicology in the late 19th century, dispelling the widely held assumption that a uniform mathematical acoustics (harmonics proper) provided the foundation of all human music-making, and demonstrating, rather, "that the Musical Scale is not one, not 'natural', nor even founded necessarily on the laws of the constitution of musical sound so beautifully worked out by Helmholtz [(1877)1954], but very diverse, very artificial, and very capricious" (Ellis 1884; 1885:526).

Newton's COLORIMETER
measuring the "celebrated *Phænomena of Colours*"

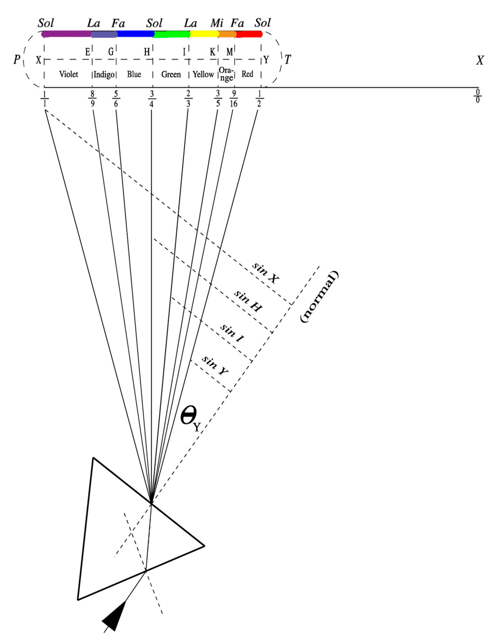
It is evident from an examination of Newton's early manuscripts that he had well-mastered canonics and tuning theory, with an emphasis on the metrical geometry of the stretched string (that is, the measured geometrical line), on the eve of (or coincident with) his initial investigations into the nature of light and colors. The central optical phenomenon that occupied Newton's attention was that of the unequal refrangibility ('breaking up,' fragmenting) of the white light of the sun when refracted through prisms. And the immediate evidence for his novel insight that [white] "*Light* itself is a *Heterogeneous mixture of differently refrangible Rays*" was the unexpected geometry of the spectral display, the perceived phenomenon of the spectrum's linear elongation.

> " … in the beginning of the Year 1666 … I procured me a Triangular glass-Prisme, to try therewith the celebrated *Phænomena of Colours*. And in order thereto having darkened my chamber, and made a small hole in my window-shuts, to let in a convenient quantity of the Suns light, I placed my Prisme at its entrance, that it might be thereby refracted to the opposite wall. It was at first a very pleasing divertissement, to view the vivid and intense colors; but after a while applying myself to considering them more circumspectly, I became surprised to see them in an *oblong* form; which, according to the received laws of Refraction, I expected should have been *circular*. …
>
> … And so the true cause of the length of that Image was detected to be no other, then, that *Light* consists of *Rays differently refrangible*, which, without any respect to a difference in their incidence, were, according to their degrees of refrangibility, transmitted towards divers parts of the wall. …
>
> … the *manner*, how colours are produced by a Prisme, is evident. For, of the Rays, constituting the incident light, since those which differ in Colour proportionally differ in refrangibility, they by their unequal refractions must be severed and dispersed into an oblong form in an orderly succession from the least refracted Scarlet [Red] to the most refracted Violet" (Newton, "New Theory about Light and Colours," 1672; *PLNP* 47-59; see **Figure 5**).

Figure 5
'Several Sorts of the Sun's Promiscuous Rays'

Newton's second paper on light and colors made most explicit the analogy between a prism's action in 'severing' sunlight into its colorific constituents and 'dividing' a sonorous string into a scale of tones:

> "Whence if the rayes which come promiscuously from the Sunn, be refracted by a Prism, … those of *severall sorts* being variously refracted must go to several places on an opposite paper or wall & so parted, exhibit every one of their owne colours, which they could not do while blended together. And because refraction onely *severs* them, & *changes* not the bigness or strength of the ray, thence it is, that after they are once well severed, refraction cannot make any further changes in their colour. … And possibly colour may be distinguished into its principal Degrees, Red, Orange, Yellow, Green, Blew, Indigo, and deep violet [= purple], on the same ground that Sound within an eighth ['octave'] is graduated into tones" (Newton "An Hypothesis Explaining the Properties of Light," 1675).

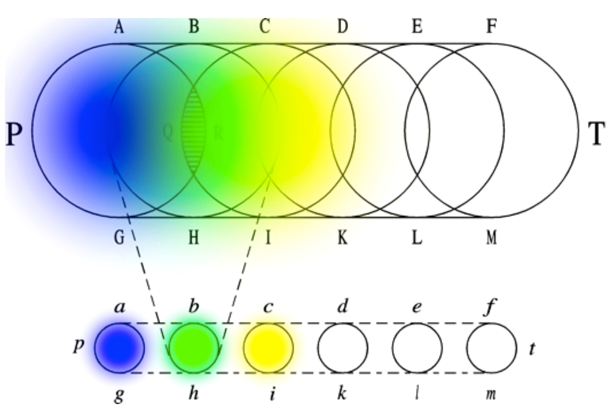


The natural 'action' or 'event,' then, of prismatic refraction (the breaking apart of white light) was understood to be the 'severing' (as in cutting, dividing, sectioning, partitioning) of the whole 'compound' of sunlight into its constituent 'parts,' and to disperse those 'colorific' parts (the 'several sorts' of colors) into an 'orderly succession' (the spectral "Image of the Sun"), according to the magnitudes of their refractions, from one extreme to the other. In much the same manner the *technē* of canonics partitions a whole tensioned and resounding string, distributing its 'sonorific' parts in an orderly succession (a *melody* or musical scale) according to the relative magnitudes of its constituent tones and intervals. In both cases what distinguishes the perceptual qualities of one color from another and one tone from another within their respective perceptual continua, is directly indexed to their places or relative positions within a metrical geometry, the orderly successions ranging from the most to least refracted visible colors and the lowest to highest tones in an 'octave.'

A phenomenon closely related to the refractive separation and dispersion of the manifold rays of white light is that of the mixing or compounding of the individually 'severed' colors. Newton demonstrated experimentally that "to the same degree of Refrangibility ever belongs the same Colour, and to the same Colour ever belongs the same degree of Refrangibility" ("New Theory About Light and Colours," 1672). And he extended that principle to clarify the oblong shape of the spectrum in terms of the mixings of individual (homogeneal) colors.

> "… it is to be considered that the Rays [of light] which are equally refrangible do fall upon a Circle answering to the Sun's Disque [subtending an angle of about ½ ° or 1/720th of a great circle]. … Let therefore **AG** [in **Figure 6**] represent the Circles which all the most refrangible Rays propagated from the whole Disque of the Sun, would illuminate and paint upon the opposite Wall if they were alone; **FM** the Circle which all the least refrangible Rays would in like manner illuminate and paint if they were alone; **BH**, **CI**, **DK**, **EL**, the Circles which so many intermediate sorts of Rays would successively paint upon the Wall, if they were single propagated from the Sun in successive order, … and conceive that there are other intermediate Circles without Number, which innumerable other intermediate sorts of Rays would successively paint upon the Wall if the Sun should successively emit every sort apart. And seeing the Sun emits all these sorts at once, they must all together illuminate and paint innumerable equal Circles, of all which, being according to their degrees of Refrangibility placed in order in a continual Series, that oblong spectrum PT is composed. … [consequently] every one of the Circles was refracted according to some most regular, uniform, and constant Law." (Newton, *Opticks* [1704]1979:38-39, 41).[24]

Figure 6
Prismatic Color-Mixing
(after Newton's diagrams)



"Primitive [prismatic] colors can be exhibited [matched to sense] by the composition of the neighboring colors on each side of them. This can be tested in various ways, exactly as in the composition of white [light], … I myself have tried some whereby golden yellow was made from saffron and pale yellow, leek green from pale yellow and sea green (or even less perfectly from golden yellow and cyan), and cyan from sea green and Indigo; and all the other colors can be compounded from the colors bordering on each side. Moreover, indigo, tempered by mixing with the extremity of the red became purple, and vermillion mingled with a touch of extreme purple turned out scarlet, just as if there was an affinity between the extremities of the colors as there is in the sounds between the termini of an octave" (Newton, *Optical Papers, Optica* II, *Lecture* 8, c. 1671-2, p. 507).

The rays of light illuminating the circles **AG** (blue) and **CI** (yellow) will overlap at the center of the circle **BH** (green). Where the circles intersect, such as "QR in the middle of BH," the combined rays generate a green, compounded of the yellow and the blue, which is fully 'like in appearance' to that central monochromatic green. Echoing Newton, 'these geneses actually take place in nature and are daily seen,' and they are evidenced "in Experiments, where Sense is Judge" (*Opticks* 1704: 123). Any given perceptible quality of color is then, according to its genesis, of "two sorts … the one original and simple [homogeneal], the other compounded of these [heterogeneal]" (Newton, "New Theory of Light and Colours," 1672). If the prismatic dispersion is 'stretched out' (as in ***pt***, **Figure 6**) individual colors, such as ***ag*** (blue) and ***ci*** (yellow) will not compound or 'alloy' ("sensible Allay") to generate intermediates ones such as ***bh*** (green), the circular areas illuminated by such rays lying at distances where they do not overlap.[25]

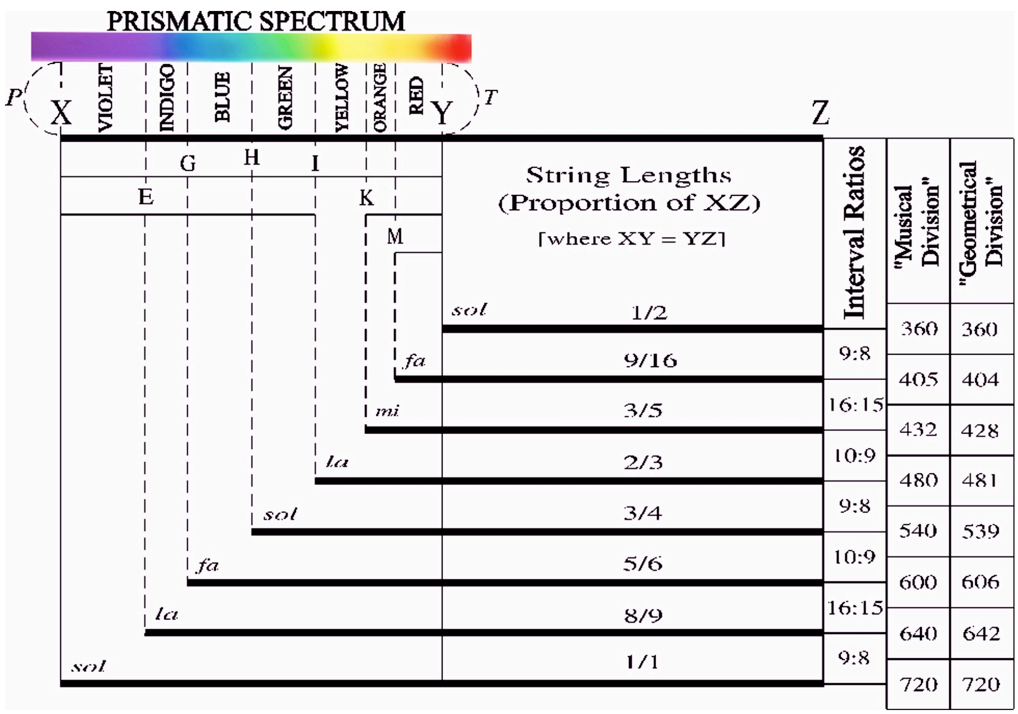
The perceptual phenomena of color-mixing were experimentally demonstrated, described and summarized into a proposition: "Colours may be produced by composition which shall be like to the Colours of homogeneal Light as to the Appearance of Colour", and Newton then introduced an Archimedean principle, 'equal weights balance at equal distances,' to account for the mixing, i.e., "for the yellow and the blue on either hand, if they are equal in quantity they draw the intermediate green equally towards themselves, and so keep it as it were in Æquilibrion, that it verge not more to the yellow on the one hand, and to the blue on the other, but by their mix'd Actions [Æquipollence] remain still a middle Colour" (Newton, *Opticks* [1704]1979:132, 133).

Increasingly, Newton's examination of the elongated spectrum (an "Image of the Sun"), the heterogeneous composition of white (sun) light, and the mixing of colors, required careful and precise measurements of the relationships (relative distances) of the various colors in their orderly lengthwise dispersion. Continuing to attend primarily to the metrical geometry of the spectrum (and eschewing all but the most general suppositions about the physical/physiological causes of the phenomenon, e.g., 'actions'), Newton proceeded empirically to "examine diligently the dimensions of each of its colors, their distances from one another."

> "… it will be worthwhile for us to investigate in more detail the shape of the colored image formed by light flowing through a narrow, round hole into a dark room and then passing through a prism, and to examine diligently the dimensions of each of its colors, their distances from one another, and also the degree of refrangibility corresponding to the individual kinds of rays. [A spectrum being projected from a prism, Newton then said,] I marked with a pen the places where the most perfect colors of their kind and their boundaries fell on a transverse paper, and having frequently repeated such observations and compared them to one another, I finally drew … conclusions one by one: … When I observed these with as much care as I could, trusting not only my own senses, but also relying on the judgment of others (because of the extreme difficulty in precisely distinguishing the colors' boundaries and places of greatest perfection), I drew the dimensions of the image according to these determinations, … so that everything turns out proportionate to the quantity of green with a more refined symmetry. … And (as I may briefly describe) everything appeared as if the parts of the image occupied by the colors were proportional to a string divided so it would cause the individual degrees of the octave to sound. … Specifically, extending the image's diminished length *XY* to *Z* so that *YZ* is equal to *XY*, imagine *XZ* to be a string that one must divide within *XY* as if the individual segments extending to *Z* are to emit the individual degrees of the octave (sol, la, fa, ut, re, mi, fa, sol) [later *sol, la, fa, sol, la, mi, fa, sol*]. This will occur by bisecting *XY* at *H* and trisecting it at *G* and *I*, and, in turn, by trisecting *XI* at *E* and taking *KY* to be the fifth and *MY* to be the eighth part of the whole *XY*. The semitones, *EG* and *KM* will represent indigo and orange, and the other five tones [intervals], *XE* [*GH*, *HI*, *IK*, and *MY*], the other five preeminent colors [violet, blue, green, yellow, red]. Each of these, when the entire mass of colors fell alike on the whole figure, was respectively contained within these individual parts. Approximately in the middle of these parts each color appeared the most brilliant and intense of its own species." (Newton, *Optical Papers*, *Optica* II, *Lecture* 11, c. 1671-2, pp. 537-549).

From careful measurements of the boundary locations of the "more eminent species" of colors, Newton's data revealed a precise 'musical-geometrical' sectioning of the oblong shape (*forma*) of the spectral "Image of the Sun" (**Figure 7**). He stated the results in a quasi-Euclidian canonical form, and expanded on that by listing the corresponding diatonic lengths of strings resounding in order, their rational 'differences' (intervals) demarking the spaces occupied by the "preeminent colors". In numbers, the lengths were given both for a "a string divided musically", — that is, "… divided after the manner of a Musical Chord, … and conceive [its parts, XZ, EZ, GZ, HZ, and so on below] to be in proportion to one another, as the Numbers, 1, 8/9, 5/6, 3/4, 2/3, 3/5, 9/16, 1/2, and so to represent the Chords of the Key, and a Tone, a third Minor, a fourth, a fifth, a sixth Major, a seventh and an eighth above that Key. And the Intervals [MY, KM, IK, HI, and so on below] will be the Spaces which the several Colours (red, orange, yellow, green, blue, indigo, violet) take up" (Newton, *Opticks* [1704]1979:128) — as well as approximately for "a string divided geometrically" (as in **TABLE 2**, above).[26]

Figure 7
Newton's Canonical Tuning of the Prismatic Spectrum of Colors
(composite summary of data)



The canonical model partitions the spectral continuum and quantifies the 'spaces' of the "more eminent colors" in terms of rational rather than arithmetic magnitudes. For example, the spaces of *Red* (*MY* = **405:360**), *Green* (*HI* = **540:480**) and *Violet* (*XE* = **720:640**) are all ratio **9:8**, with a equal-12 logarithmic value of $(12/\log 2)(\log 9 - \log 8) = 2.039100017\ldots$ signifying its relative magnitude (that is, **2+** of the **12** equal-ratio parts of the whole). Also, the spaces occupied by *Blue* and *Yellow* measure the same (ratio **10:9**), as do those for *Indigo* and *Orange* (ratio **16:15**). The musical-metrical geometry accommodates not only the apparent boundaries between the "more eminent colors", whose visible extents are unequal, but also the orderly succession of all the intermediate colors and tones within given limits (least to most refracted, longest to shortest string-length). The partitioning, empirically grounded and diligently measured, also incorporated Newton's basic concept of the mixing of adjacent colors, so that "in the middle of these parts each color appeared the most brilliant and intense of its own species." The geometry and measurements affirm the spectral dispersion of white light and colors to be a thoroughly symmetrical and rationally systematic phenomenon.

None of Newton's principal 'animadversors,' such as Robert Hooke [1635-1703] or Christiaan Huygens [1629-1695], objected to this musical-geometrical description of the colorific spectrum — they were also practitioners of canonics and avidly engaged in their own audial-visual, acoustical-optical rhetorics (see Kassler and Oldroyd 1983; Huygens [1691]1986). But they were opposed to the implications of the infinite divisibility — and hence the idea of innumerably many colors (light rays) in the composition — of the white light of the sun which, in appearance, is *uniform* and *homogeneous* rather than *difform* and *heterogeneous* (see especially Shapiro 1973; 1979; Sabra 1981; Sepper 1994). In the "New Theory about Light and Colours" (1672, *PLNP* 1958:47-59), Newton wrote that …

> "Light itself is a *Heterogeneous mixture of differently refrangible Rays,* … Nor are there only Rays proper and particular to the more eminent colours, but even to all their intermediate gradations … To the same degree of Refrangibility ever belongs the same colour, and to the same colour ever belongs the same degree of Refrangibility … And this Analogy 'twixt colours, and refrangibility, is very precise and strict; the Rays always either exactly agreeing in both, or proportionally disagreeing in both … But the most surprising and wonderful composition was that of *Whiteness.* … the usual colour of *Light*; for, Light is a confused aggregate of Rays indued with all sorts of colours; as they are promiscuously darted from the various parts of luminous bodies."

These assertions not only contradicted Hooke's theory that there were no more than two kinds of colors, but they also threatened some cherished assumptions about proper scientific procedure, so Hooke objected:

> "... that there are an indefinite variety of primary or original colours, amongst which are yellow, green, violet, purple, orange, &c. and an infinite number of intermediate gradations, I cannot assent thereunto, as supposing it wholly useless to multiply entities without necessity, since I have elsewhere shewn [*Micrographia* 1665], that all the varieties of colours in the world may be made of two [blue and red] … [the other colors] intermediate are the dilutings and intermixtures of those two …" ("Mr. [Robert] Hooke's Considerations... ," 1672, *PLNP* 1958:113).

Hooke's accusation that Newton had violated a principle of scientific reasoning (parsimony) was, if not mistaken, simply irrelevant. Newton was not in basic disagreement with the dictum (originally medieval) that 'plurality is not to be posited without necessity,' or a presumption that nature is 'simple' and hence that its explanations need be economical and succinct. But in the "New Theory" Newton was not engaged in hypothetical posturing or argument from 'principles' founded on metaphysical stances. He was, rather, reporting (and then refining) a fact grounded in quotidian human experience, that of the manyness or plenitude (the diverse variety) of colors, according to one's direct acquaintance with the "*Phænomena* of *Colours*". In contrast to fantastic 'hypotheses' dominating explanations (and to keep first things first), the interpretative process that Newton favored can be said to operate 'backwards,' that is, knowledge or understanding is advanced by 'proceeding' by inference from perceived 'effects' (such as the manifold variety and natural orderliness of spectral colors) to suspicions as to their 'causes' (such as generative processes, the 'refrangibilility' and 'original heterogeneity' of light), and on to experimental procedures to confirm or infirm those suppositions.

"As in Mathematicks, so in Natural Philosophy, the Investigation of difficult Things by the Method of Analysis, ought ever to precede the Method of Composition. This Analysis consists in making Experiments and Observations, and in drawing general Conclusions from them by Induction, and admitting of no Objections against the Conclusions, but such as are taken from Experiments, or other certain Truths. For Hypotheses are not to be regarded in experimental Philosophy. … By this way of Analysis, we may proceed from Compounds to Ingredients, and from Motions to the Forces producing them; and in general, from Effects to their Causes, …" (Newton, *Opticks* [1704]1979:404).

Newton was among those 17th century …

"... thinkers who sought to flesh out in the context of experimental science our understanding of *induction*. This general process whereby the mind forms from its commerce with sensible nature *ideas* about how that nature works had of course been recognized and set in contrast to the syllogism (deductive reasoning) from the earliest times, ... That there is such a process is evident from the fact that we are not born with a predetermined set of notions about the world, but develop our concepts first from experience. ... The most fertile development for semiotics in this area of logic comes with the re-discovery by C. S. Peirce around 1866 that the notion of induction is heterogeneous, comprising not one, but two distinct species of movement: the movement of the mind whereby we form an hypothesis on the basis of sensory experience, which Peirce called *abduction* (sometimes 'hypothesis', also 'retroduction' [also 'abductive inference,' 'guessing']), and the movement back whereby we confirm or infirm our hypothesis with reference to the sensory, for which movement Peirce retained the name *induction*" (Deely 1982:68-69, 71).

Hooke and Newton differed in their styles of thought, methods and philosophical assumptions, and even as to what were the salient facts to be addressed in natural science inquiries, all of which contributed to perpetual bickering and lack of mutual understanding (Sabra 1981; Mamiani 2000). Nevertheless their exchanges over the nature of light and colors were replete with informative acoustical and musicological analogies and metaphors. Newton answered Hooke, publically and at length:

"I knew that the *Properties* of *Light* were in some measure capable of being explicated by many different Mechanical *Hypotheses*. And therefore I chose to decline them all, and to speak of Light in *general* terms, considering it abstractly, as something or other propagated every way in straight lines from luminous bodies, without determining, what that Thing is; … And for the same reason I chose to speak of *Colours* according to the information of our Senses, as if they were Qualities of Light *without* us" (Newton, "Mr. Newton's Answer to Some [Robert Hooke's] Considerations upon his Doctrine of Light and Colors," 1672).

Newton also wrote that Hooke made "…but two *species* of [colors], accounting all other Colors in the world but various degrees and dilutings of those two"; and of this and other of Hooke's "…Explications, I think it would be no difficult matter to shew, that they are not only *insufficient*, but in some respects to me (at least) *un-intelligible*." … And there is as great a difficulty in the Number of Colours." Hooke had demonstrated his two-color theory using translucent wedge shaped boxes filled with colored fluids (red and blue liquors). 'White' light shining through the boxes appeared as different colors at different thickness of the wedges, even while the tinctures were of but one sort or the other. Newton considered those 'experiments' to be defective both in design and in the inferences drawn from the observations.

In addition to the demonstrations using translucent boxes, Hooke had enlisted a musical/acoustical metaphor in his criticism of Newton's theory:

"But why there is a necessity, that all those motions, or whatever else it be that makes colours, should be originally in the simple rays of light, I do not yet understand the necessity of, no more than all those sounds must be in the air of the bellows, which are afterwards heard to issue from the organ-pipes; or in the string, which are afterwards by different stoppings and strikings produced; which string (by the way) is a pretty representation of the shape of a refracted ray to the eye; for the ray is like the string, strained between the luminous object and the eye, and the stop of fingers is like the refracting surface, on the one side of which the string hath no motion, on the other a vibrating one. Now we may say indeed and imagine, that the rest or straightness is causes by the cessation of motion, or coalition of all vibrations; and that all the vibrations are dormant in it; but yet it seem more natural to me to imagine it the other way" ("Mr. [Robert] Hooke's Considerations... ," 1672, *PLNP* 1958:111).

Newton countered with a different musical/acoustical metaphor:

"Now, if the *Animadversor* [Hooke] contend, that all the Reds and Yellows of the one Liquor, or Blews and Indigo's of the other, are only various degrees and dilutings of the same Colour [say a Tincture of *Aloes*], and not diverse colours, that is a Begging of the Question: And I should as soon grant, that the two *Thirds* or *Sixths* in Musick are but several degrees of the same sound, and not diverse sounds. Certainly it is much better to believe our Senses, informing us, that *Red* and *yellow* are diverse colours, and [then] to make it a Philosophical *Quære*, Why the same Liquor doth, according to its various thickness, appear of those diverse colours, than to suppose them to be the same colour because exhibited by the same liquor?"

In the musicological jargon of the times, and in Newton's own studies (the "discourse … of strings sounding" and the essay "Of Musick"), there are two sizes of "Thirds", a 3rd major (ratio 4:5) and a 3rd minor (ratio 5:6), and two sizes of "Sixths", a 6t major (ratio 3:5) and a 6t minor (ratio 5:8). The ordinal nomenclature was conventional, and it derived from the prevalent musicological custom of theorizing in terms of 'diatonically' (rather than 'chromatically') ordered scales. The terminology tended to treat 'major' and 'minor' intervals as slight modifications of one another; indeed, for some musicological purposes the quantifiably and acoustically different magnitudes of certain intervals, such as a 2nd major (ratio 9:8) and a 2nd minor (ratio 10:9), were simply ignored (see, for example, Playford 1667). But here Newton rejects the nominalist implication that because various "Thirds" or "Sixths" are named the same they are the same in fact. For both sounds and lights, tones and colors, it is neither names nor metaphysical 'forms' that determine their character but the information of one's senses and experiences.

"But if there be yet and doubting, 'tis better to put the Event on further Circumstances of the Experiment, "than to acquiesce in the possibility of any *Hypothetical* Explication" (Newton).

For Newton, "*Whiteness* … the usual colour of *Light*" was the epitome of the *blending* (compounding or mixing) of innumerably many different rays of light into a single, uniform and unique appearance to sense. "Whiteness is ever compounded and the most so of all the colours … for, Light is a confused aggregate of Rays indued with all sorts of colours". And so, answering Hooke, Newton listed a plethora of ways of producing 'whiteness by mixtures,' some of which continued to invoke musical/acoustical comparisons. For example, of the mixed lights reflected from multicolored objects …

"… a perfect and *intense* whiteness is not to be expected, but rather a color between those of light and Shadow, or such a Gray or Dirty colour as may be made by mixing White and Black [pigments] together. … there may also be produced the like Dirty colour by mixing several *Painter's colours* together, And the same may be effected by Painting a Top (such as Boys play with) if diverse colours. For, when it is made to circulate by whipping it, it will appear of such a dirty color."

A rapidly spinning particolored *top*, which effects a visual blending of diverse colors into the singular appearance of a 'dirty white,' was a common trope throughout the history of research into vision and optics. Plato, in *Timaeus* 67*d*-68*d*, had discussed the complex problem of mixing or blending 'colors':

"… color being a flame that flows from individual bodies and having parts proportionate to our vision so as to produce a sensation … [a visual stream flows outward from the eye to comingle with one flowing inward from objects, and] … all manner of colors are born through this melee; and we call this affection *dazzling*, while to that which produces it we give the names *brilliant* and *glittering*. Again, when the kind of fire is between the extremes of *white* [appearance] and *dazzling* [object] reaches to the moist part of the eyes and is blended with that part, then it isn't *glittering*, but, because the fire's ray shines through the moisture it's mixed with, it produces a bloody color, which we call by the name of *red*. And *brilliant* mixed with *red* and *white* becomes *yellow-orange*. But to say in what proportions these have been mixed, even if someone knew, doesn't make sense, since one wouldn't be able, even moderately, either to state any necessity for it or give the likely account. … But, if investigating these matters, someone were to make a test of all this through experiment, he would only show his ignorance of the difference between the human and the divine nature: that it is god who is sufficiently knowledgeable, and also able, to blend together the many into a one and again in turn to dissolve a one into a many, but no one among humans either is now of ever will be in the future sufficient for either of these."

While Plato had not discerned any regular relationships between perceived colors ('affections') and the objective conditions that give rise to them, in the contemporary harmonic and canonical theories, where the correlations of perceived sounds (tones) and the variable magnitudes of tensioned strings were well-established, the audial/acoustical 'blending' of diverse sounds and tones proportionally into singular qualities was the very definition and distinction of 'consonance' and 'dissonance.'

"… one must investigate all the sounds which, being either swift or slow, appear either shrill or deep, sometimes dissonant as they course along because of the dissimilarity of the motion they produce in us, sometimes consonant because of their similarity. … out of shrill [higher] and deep [lower] they blended together a single affection" (Plato, *Timaeus* 80*a-b*).

"And we know of [different] notes that some are consonant, some dissonant; and the consonant notes make a single blend [*krasis*] from both notes, but dissonant notes do not. This being the case, it is reasonable that consonant notes, since they make a single blend [*krasis*] of sound for both notes, are related numerically to one another in one name, …" (Euclid, *Sectio canonis*, Introduction).

"We enjoy concord because it is a blend [*krasis*] of opposites [that is, differents] which have a ratio to one another" (Aristotle, *Problems* XIX, 38; see also Sorabji 1972).

In addition to the audial phenomenon of 'blending,' different notes or sounds had long been regarded as 'parts' of some whole sonorous tone, that is, 'each sound consists of many sounds.' Following Ptolemy's lead (*Harmonics*, *Optics*), Boethius re-stated the notion for both visual/optical and audial/acoustical *blending* and 'consonance.'

"One should not think that when a string is struck, only one sound is produced, or that only one percussion is present in these numerous sounds; rather, as often as air is moved, the vibrating string will have struck it. But since the rapid motions of sounds are connected, no interruption is sensed by the ears, and a single sound, either low or high, impresses the sense. Yet each sound consists of many sounds, the low of slower and less frequent sounds, the high of faster and more frequent ones. It is as if someone fashions a cone—which people call a 'top'—and applies one stripe of red or some other color to it and spins it as fast as possible, the whole cone seems dyed with the red color, not because the whole thing is thus, but because the velocity of the red stripe overwhelms the clear parts, and they are not allowed to appear. … For consonance is the concord of mutually dissimilar pitches brought into one" (Boethius, *Fundamentals of Music*, 6th century CE, 1989:11-12; see also Creese 2010: 235-240).

The concept of the perceptual 'blending' (of multiple lights or of multiple sounds) was central to Newton's conjectures about the similarities of visual and audial 'harmonies,' even while he acknowledged the limitations of such analogies. Newton valued the "musicall" description of the geometrical shape of the spectrum, which incorporated his notions about mixing spectral colors (**Figure 6**, **Figure 7**), …

"… because it agrees with the phenomena [of refracted spectral colors] very well" [and] "…because it perhaps involves something about the harmonies of colors (such as painters are not altogether unacquainted with, but which I myself have not yet sufficiently studied), … perhaps analogous to the concordance of sounds. It will even appear more probable by noting the affinity existing between the outermost purple and red, the extremities of the colors, such as is found between the ends of the octave (which can in a way be considered as unisons)" (Newton, *Optica II*, "Lecture 11," c. 1671-2).

"In his 'Hypothesis' of 1675, Newton offered the examples of the harmony of 'golden & blew' and the discord of 'red & blew' … His most comprehensive account of color harmony, however, was in a draft for a projected Prop. 12 for Bk. II of the *Opticks* [of 1704]: 'For instance green agrees with neither blue nor yellow for it is distant from them but a note or tone above & below[.] Nor doth Orange for the same reason agree with yellow or red: but Orange agrees better w^th an Indigo blew then w^th any other colour for they are fifts. And therefore painters to set off Gold do use to lay it upon such a blew. So red agrees well w^th a sky coloured blew for they are fifts & yellow w^th Violet for they are also fifts. But this harmony & discord of colours is not so notable as that of sounds because in two concord sounds there is no mixture of discord ones, in two concord colours there is a great mixture, each colour being composed of many others' ([Newton] *Add. Ms*. 3970, ff. 348-9)" (Shapiro, note 27 in Newton, *Optical Papers*, 1984:546).[27]

During the 17^th century, scientific research had confirmed the heterogeneous motions of a single stretched string (see Mersenne 1636-7; Dostrovsky 1975; H. F. Cohen 1984). A vibrating string moves, at once, in all its integral parts, and so it moves the diverse parts of the surrounding air generating, at once, a 'multiplex' or 'compound' sound consisting of a 'fundamental' frequency along with its integral multiples (its 'overtones, 'harmonics' or 'partials'). Newton noted this phenomenon in his student notebooks, attributing the propagation of such 'complexes' of sound to the heterogeneous composition of air.

"In every sound the eighth above it …seems to be heard. For there is some more subtle, some more gross matter in the air and the subtlest matter is prone to quickest vibrations [and the grossest matter the slowest], though the motion of both proceed from the same cause (as in the vibration of a string or pipe); thus twigs vibrate after the branches. … Hence each [fundamental] sound has its concomitant eight [octave], and perhaps 15^th [double octave] and 22^nd [quadruple octave] to a good ear, above …it. … And more violent breathing raises the sound an eighth or 15^th, … or but seldom to a 12^th [octave + fifth]" (Newton, in McGuire and Tamny [1983]2002:389-391).[28]

Newton's replies to Hooke did not put the puzzles over colorific 'manyness' or 'fewness' to rest, and Christiaan Huygens carried on:

"Me thinks, that the most important Objection, which is made against him [Newton] by way of *Quære*, is that, Whether there be more than two sorts of Colours. For my part, I believe, that an Hypothesis, that should explain mechanically and by the nature of motion the Colors Yellow and Blew, would be sufficient for all the rest, in regard that those others, being only more deeply charged (as appears by the Prismes of Mr. Hook.) do produce the dark or deep-Red and [dark] Blew; and that of these four all the others may be compounded. Neither do I see, why Mr. Newton doth not content himself with the two Colors, Yellow and Blew; for it will much more easy to find an Hypothesis by Motion, that may explicate these two differences, than for so many diversities as there are of others Colors. ... As for the composition of White made by all the Colors together, it may possibly be, that Yellow and Blew might also be sufficient for that..." (Christiaan Huygens, "An Extract of a Letter lately written by an ingenious person from Paris containing some Considerations upon Mr. Newton's Doctrine of Colors," *Philosophical Transactions*, No. 96, 21 July 1673, pp. 6086-6087; *PLNP* 1958:136-137).

In responding to Huygens, Newton restated his disregard for any 'Hypothesis by Motion,' (whether of two colors or not), and he re-expressed more clearly his own ideas, elaborating the basic themes of his initial doctrine. It is notable that he continued to discern that the 'disputes' were arising less from 'facts and experiments' than a reluctance to accept "the information of our Senses" along with problems of language in respect of that information — difficulties with core concepts that were shared or not in the community of discourse. In both the formation and communication of scientific ideas, "the nature of language [was] the crucial problem in the epistemology of the new science" (Aarsleff 1982:251). Consequently, Newton answered Huygens with clarified 'definitions' and derived 'propositions' — "Mr. *Newtons* Answer to the foregoing Letter [from Huygens] further explaining his Theory of Light and Colors, and particularly that of Whiteness …" (*Philosophical Transactions,* No. 96, 21 July 1673, pp. 6087-6092; *PLNP* 1958:137-142):

Definitions

1. I call that Light *homogeneal*, similar or uniform, whose rays are equally refrangible.
2. And that *heterogeneal*, whose rays are unequally refrangible. Note. There are but three affections of Light in which I have observed its rays to differ, vis. Refrangibility, Reflexibility, and Colors; and those rays which agree in refrangibility agree also in the other two, and therefore may be defined homogeneal, especially since men usually call those things homogeneal, which are so in all qualities that come under their knowledge, though in other qualities that their knowledge extends not to there may possibly be some heterogeneity.
3. Those colors I call simple, or homogeneal, which are exhibited by homogeneal light.
4. And those compound or heterogeneal, which are exhibited by heterogeneal light.
5. Different colors I call not only the more eminent species, red, yellow, green, blew, purple, but all other the minutest gradations; much after the same manner that not only the more eminent degrees in Musick, but all the least gradations are esteemed different sounds.

Propositions

1. The Sun's light consists of rays differing by indifinite degrees of Refrangibility.
2. Rays which differ in refrangibility, when parted from one another do proportionally differ in the colors which they exhibit. These two propositions are matter of fact.
3. There are as many simple or homogeneal colors as degrees of refrangibility. …
4. Whiteness in all respects like that of the Sun's immediate light and of all the usual objects of our senses cannot be compounded of two simple colors alone. …
5. Whiteness in all respects like that of the Sun's immediate light cannot be compounded of simple colors without an indefinite variety of them. …

   …

10. The Sun's light is an aggregate of an indefinite variety of homogeneal colors; … And hence it is, that I call homogeneal colors also primitive or original. And thus much concerning colors."

Newton had again insisted on an "indefinite variety" of spectral (simple or "primitive") colors, original and connate with the light of the sun. And 'Definition 5' was especially clear in its comparison of the "whole mass of colors" of the spectrum with a whole tensioned string and the respective partitioning of each which ranged from the gross and general sorts ("eminent species") to the most subtle "minutest" and "least gradations". The idea of such 'levels of scale' simulated, roughly, the levels of 'diatonic' and 'chromatic' divisions of an 'octave,' that is, those containing, at once, larger (and fewer) intervals as well as smaller (and more plentiful) intervals (as in **TABLE 1**). In his essay "Of Musick" (c. 1665, fols. 138^r-v^) Newton wrote: "The prime ℘ts of an 8^th^ are a 5^t^ & 4^t^; of a fift are a 3^rd^ major and a 3^rd^ minor; which two consist y^e^ first of a tone major & a tone minor, y^e^ 2^d^ of a tone major & semitone. So y^t^ an eight consists of three tone majors [**3T**], two tone minors [**2t**] and two semitones [2S]". The observation referred to a *first order* and rational partitioning resulting in a set of *difform* (unequal) "musicall" intervals ("prime ℘ts"), similar to the manner in which angular degrees are *first* divided into small parts ('minutes'), and those then *secondarily* divided in yet smaller ones ('seconds'). "An Eight is first divided into a 5^t^ & 4^t^ … An 8^th^ is next divided into a third major and a 6^t^ minor [the complements 4:5 and 5:8], and lastly into a 3^d^ minor & 6^t^ major [the complements 5:6 and 3:5]" (Newton, "Of Musick", fol. 138^r^). Most of these gross discrete intervals (*ratios*), those Newton especially referred to as "musicall", can be illustrated by simple *rational* interpolations:

1		:			2	8th (octave, ratio 1:2)
2	:	3	:		4	5t (fifth, ratio 2:3) & 4t (fourth, ratio 3:4)
4 : 5 : 6						3rd major (ratio 4:5) & 3rd minor (ratio 5:6)
8 : 9 : 10						2d major (ratio 8:9) & 2d minor (ratio 9:10)
8		:	15	:	16	7th major (ratio 8:15) & semitone (ratio 15:16)

Further rational divisions are shown in **TABLE 1**. And it need be emphasized that these 'bisections' result in *unequal* parts; each given interval (such as **2:4**) consists of two complementary intervals, one greater (**2:3**) and one lesser (**3:4**) in size. But Newton had also considered (in his "discourse … of strings sounding" c. 1665) an endless variety of "geometrically progressionall" partitions of the continuous magnitude of a tensioned string; those were expressed "equidistant" intervals by interpolating indefinitely many mean proportionals, $1{:}2^{n/x}$ (**TABLE 3**). Similarly, he alternately characterized the geometry of the oblong spectral continuum as composed of a selection of "eleven mean proportionals" (**Figure 7**) as well as "innumerable equal circles" illuminated by an orderly succession of rays of light "without Number" (**Figure 6**). The abstract ideas of hierarchy and successional order in the canonical partitioning of continuous magnitudes, grounded in a 'perceptual acquaintance' (Yolton) and 'commerce with sensible nature' (Deely), were integral to Newton's theory of light and colors:

> "By the Rays of Light I understand its least Parts, and those as well Successive in the same Lines, as Contemporary in several Lines. For it is manifest that Light consists of Parts, both Successive and Contemporary" (Newton, "Definition 1," *Opticks* [1704]1979:1).

A few months later, Newton again took up Huygens's question about the "Number of Colors", continuing to insist on the validity of his own definitions, the reasonings therefrom, and warranting them by further references to natural phenomena. What Newton called 'the analogy of nature' was the notion that "all natural things", the "usual objects of our senses", occurred with "indefinite variety", insofar as each was compounded or composed of diverse variables (continuous magnitudes).

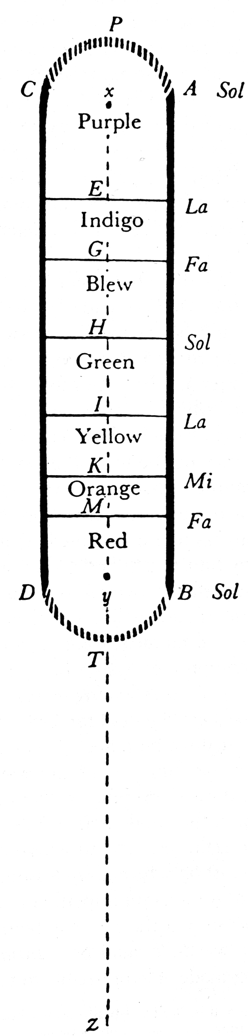
> "As to the Contents of his [Huygens's] Letter, I conceive, my former Answer to the *Quære* about the *Number* of *Colors* is sufficient, which was to this effect: That all colors cannot practically be derived out of the *Yellow* and *Blew*, and consequently that those Hypotheses are groundless which imply they may. If you ask What colors cannot be derived out of *yellow* and *blew*? I answer, none of all those which I defin'd to be Original; and if he can shew by experiment, how they may, I will acknowledge my self in an error. Nor is it easier to frame an *Hypothesis* by assuming only two Original colors rather than an indefinit variety; unless it be easier to suppose, that there are but two figures, sizes and degrees of velocity or force of the Æthereal corpuscles or pulses, rather than an indefinit variety; which certainly would be a harsh supposition. No man wonders at the indefinit variety of Waves of the Sea, or of sands on the shore; but, were they all but two sizes, it would be a very puzzling *phænomenon*. And I should think it as unaccountable, if the several parts or corpuscles, of which a shining body consists, which must be suppos'd of various figures, sizes and motions, should impress but two sorts of motion on the adjacent Æthereal *medium*, or any other way beget but two sorts of Rays. But to examine, how Colors may be explain'd *hypothetically*, is besides my purpose. I never intended to shew, wherein consists the Nature and Difference of colors, but only to shew, that de facto they are Original and Immutable qualities of the Rays which exhibit them; and to leave it to others to explicate by mechanical *Hypotheses* the Nature and Difference of those qualities: which I take to be no difficult matter" (Newton, "An Extract of Mr Isaac Newtons Letter … concerning the Number of Colors, and the necessity of mixing them all for the production of White…" *Philosophical Transactions*, No. 97, 6 October 1673, pp. 6108-6112; *PLNP* 1958:143-146).

In whatever way Newton's meanings and definitions were (or not) being understood, much less accepted, he ventured once more, in "An Hypothesis Explaining the Properties of Light" (1675), to justify and distinguish his account from others. He *was not*, he said, to presume any peripatetic qualities, instantaneous propagations of particles or vibratory motions (with or without a medium), or make claims about any spiritual or mechanical actions. He *was*, he said, going to explore a general supposition (an 'abductive inference') about the nature of light and colors, one that was contingent upon a vast array of phenomenological facts of natural philosophy — electric effluvia, physical and physiological fluids, solar heating, purification, compression of tadpoles, things 'unsociable made sociable' by chemical processes, and so on:

> "…whatever Light be, I would suppose, it consists of Successive rayes differing from one another in contingent circumstances, as bignes, forme or vigour, like as the Sands on the Shore, the waves of the Sea, the faces of men, & all other naturall things, it being almost impossible for any sort of things to be found without some contingent variety" (Newton, "An Hypothesis," *Correspondence* 1: 370).

This supposition was presented alongside another caveat about the difference between some 'manner of speaking' and the actual phenomena spoken about: "yet while I am describing this, I shall sometimes to avoyde Circumlocution and so represent it more conveniently speak of it as if I assumed it & and propounded it to be believed". Newton then proceeded "to explain colours" according not only to the multi-variable physical contingencies of 'all natural things' but especially in reference to the canonical criteria of the "discourse of y^{e} motion of strings sounding" (c. 1665).

> "Thus much of refraction, reflexion, transparency & opacity. And now to explain colours: I suppose, that as bodyes of various sizes, densities, or tensions do by percussion or other action excite sounds of various tones & consequently vibrations in the Air of various bignesse, so when the rayes of Light by impinging on the stif refracting Superficies excite vibrations in the æther, those rayes, whatever they be, as they happen to differ in magnitude, strength, or vigour, excite vibrations of various bignesses; the biggest, strongest or most potent rayes, the largest vibrations & the others shorter, according to their bignesse, strength or power, And therefore the ends of the Capillamenta of the optique nerve, wch pave of face the Retina, being such refracting Superficies, when the rayes impinge upon them, they must there excite these vibrations, wch vibrations (like those of Sound in a trunk or trumpet,) will run along the aqueous pores of Crystalline pith of the Capillamenta through the optic Nerves into the (wch Light itself cannot doe.) & there I suppose, affect the sense with various colours according to their bignesse & mixture; the biggest with the strongest colours, Reds & Yellows; the least with the weakest, blews & violets; the midle with green, & a confusion of all, with white, much after the manner, that in the sense of Hearing, Nature makes use of aërial vibrations of severall bignesses to generate Sounds of divers tones, for the Analogy of Nature is to be observed. And further, as the harmony and discord of Sounds proceed from the proportions of the aereall vibrations; so may the harmony of some colours, as of a Golden & and blew, & and the discord of others, as of red & blew proceed from the proportions of the æthereall. And possibly colour may be distinguished into its principall Degrees, Red, Orange, Yellow, Green, Blew, Indigo, and a deep violet, on the same ground, that sound within an eighth is graduated into tones. {*}For, some years past, the prismatique colours being in a well darkened roome cast perpendicularly upon a paper about two & twenty foot distant from the Prism: I desired a friend to draw with a pencil lines crosse the Image or Pillar of colours where every one of the seven aforenamed colours was most full and brisk, & also where he judged the truest confines of them to be, whilst I held the paper so that the said Image might fall within a certain compass marked on it. And this I did, partly because my owne eyes are not very critical in distinguishing colours, partly because another, to whome I had not communicated my thoughts about this matter, could have nothing but his eyes to determin his fancy in makeing those marks. This observation we repeated diverse times, both in the same &



divers days to see how the marks on severall papers would agree, & comparing the Observations, though the just confines of the colors are hard to be Assigned, because they passe into one another by insensible gradation; yet the *differences* of the Observations were but little, especially toward the red end, & takeing meanes between those differences that were the length of the image (reckoned not by the distance of the verges of the Semicircular ends [*PT*] but by the distance of the Centers of those Semicircles [*xy*], or length of the straight sides as it ought to be) was divided in about the same proportion that a String is, between the end & the midle, to sound the tones in an eight.{*} You will understand me best by viewing the annexed figure [herein], in which *AB* & *CD* represent the straight sides about ten inches long, *APC* & *BTD* the Semicircular ends, *x* & *y* the Centers of those Semicircles, *xz* the length of a Musical String double to *xy*, & divided between *x* & *y*, so as to sound the tones exprest at the side, (that is *xH* the half, *xG* & *GI* the third part, *yK* the fift part, *yM* the eighth part, and *GE* the ninth part of *xy*;) and the intervals between these divisions express the spaces wch the colours written there took up, every colour being most briskly specific in the midle of these spaces." (Newton, "An Hypothesis Explaining the Properties of Light," *Correspondence* I: 376-377; on the section marked *…* see NOTE 33, below).[29]

The "Pillar of colours" diagram was a re-statement of the 'diatonic' relationships (the "prime ℘ts of an octave" or "principall Degrees" of color) shown above in **Figures 5** and **7**). It described the gross appearance of the oblong spectrum, in the fational terms of a musical-metrical geometry, that Newton had achieved "some years past" (in *Optica* II, *Lecture* 11, c. 1671-2); it was again presented in *Opticks* ([1704]1979:126-128). Most importantly, the lengthy "Hypothesis" statement "to explain colours" invoked the principles of the *kanōn* throughout. It began by alluding to three co-present (or co-active) continuous variables ("sizes, densities, or tensions") upon which the sound (tone and pitch) of a 'musical string' (an actual material 'body') is contingent. These are now generally known as 'Mersenne's laws' after their formalization and demonstration by Marin Mersenne (*Harmonie universelle*, 1636-7). The first variable was the ancient 'Euclidian' principle pertaining to the metrical geometry of the line/string (see **Figure 1**) — on which most of Newton's early (c. 1665) musicological research was founded — that, of a tensioned and resounding string its fundamental frequency of vibration (or perceived pitch) varies (1) inversely proportional to string-length ('size').[30] The second variable was that the frequency of vibration is (2) inversely proportional to a string's mass per unit length ('density'). And the third factor was that frequency of vibration varies (3) directly proportional to the square root of the tensioning force ('tension') on the string. Consequently, the fundamental frequency (perceived pitch) of a string in motion was understood to be contingent upon multiple co-present variables of 'the body'. This complex variety is then conveyed to the complex motions of air ("aereall vibrations") and further propagated through the nerves into the *sensorium*, a place where sensory events take place and are processed.

In the long passage "to explain colours" Newton constructed a description for the rays of light parallel to that of sounds: complex "aethereall vibrations" of rays of light "whatever they be, as they happen to differ in magnitude, strength, or vigour, excite vibrations of various bignesses". And those (three) variable and heterogeneous motions are similarly propagated, neurologically, into the sensorium, there to be identified, interpreted, characterized, combined and compared with others 'proportionally' ("…& there I suppose, affect the sense with various colours according to their bignesse & mixture; the biggest with the strongest colours, Reds & Yellows; the least with the weakest, blews & violets; the midle with green, & a confusion of all, with white, much after the manner, that in the sense of Hearing, Nature makes use of aërial vibrations of severall bignesses to generate Sounds of divers tones").

Newton's characterization of neurophysiological processes was considerably more sketchy and general than that of the eminent clinical physician and neurologist Thomas Willis [1621-1675]. And

Newton did not, as Willis did, explore the linkages between particular anatomical structures and neurological or cognitive functions (see especially Frank 1990; Kassler 1995; Rousseau 1989, 1990, 2004; Wallace 2003). Newton's idea, at best, avoided claiming any particular psycho-physical causality, in favor a simply demonstrating that significant correlations obtained between the physical (and neurophysiological) variables of 'bignessess' and the perceptual variables of colors or tones. Mechanical or metaphysical 'causes' were eschewed and those antiquated notions were replaced by that of "the information of our Senses", whereupon cognizance of the world first emerges — "The Organs of Sense are not for enabling the Soul to perceive the Species of Things in its Sensorium, but only for conveying them thither" (Newton, *Opticks* [1704]1979 403).

The 'identity' or 'character' of perceived things ensues from their becoming 'present to mind,' even while that process was not well understood. Objects themselves are not mentally present, nor do their physical 'emanations' or 'images' bear any inherent relationship to those objects — "For since the things the mind contemplates are none of them, besides itself, present to the understanding, it is necessary that something else, as a sign or representation of the thing it considers, should be present to it; and these are *ideas*" (John Locke, *Essay Concerning Humane Understanding* 4.21.4 9 [1690]). Things are 'present to mind' as signs (signals, signifiers) requiring interpretation, and, in the incipient cognitive psychologies of the 17th century, the simple *idea* of a thing, say any particular audible tone or visible color, was tantamount to the simple *perception* of that thing. And such initial sensations/perceptions were the basis of more complex and abstract ideas/perceptions; they constituted a sort of aesthetic infrastructure of thought and reasoning (see Locke [1690]1979; Deely 1982; Yolton 1984; Dalgarno [1661]2001; Wallace 2003).

Newton's parallel audial/visual "Hypothesis" ranged across considerations of physical acoustics (Mersenne's laws) through neurophysiological acoustics and psychological acoustics, to cognitive/cultural acoustics (audial perceptions and musicological concepts). The last was summarized in the statement that "possibly colour may be distinguished into its principall Degrees, Red, Orange, Yellow, Green, Blew, Indigo, and a deep violet, on the same ground, that sound within an eighth is graduated into tones", so that the elongated geometrical shape of the prismatic spectrum was completely described, graphically and verbally, by the measured boundaries and centers of those same "aforenamed colours" (corresponding to a symmetrical pattern of musical intervals, **TStTtST**).

Overall, Newton's terminology included more than 100 distinct color terms, whose differences in hue and brightness could be (and frequently were) indexed to specific positions in the spectral distribution, e.g., scarlet, vermillion, and red amongst the reds, and leek green (*porracea*), green (*viridis*), sea green (*thalassinum*) among the greens, and varieties of whites, blews, purples, yellows, etc., such as willow green, citrine yellow, bluish white, least reddish purple, as well as other colors: cyan, gray, and so on. He commanded an extensive, rich and subtle 'chromatic' vocabulary even before determining the musical-metrical geometry of the spectral dispersion (see especially *Optical Lectures*, p. 461). His terminological usages varied from highly particularistic descriptors to general types or categories; the distinctions were noted in consistent references, respectively, to "primary" colors and "principall" colors, that became increasingly important in Newton's debates and defenses.

The relatively few colors Newton called the "principall degrees" or "more eminent species" were, in most respects, general types or what is now considered to be *color categories,* which are subject to cognitive, linguistic, and cultural variability somewhat independently from the physical properties of light or the neurological/psychological processes of sensation/perception as such. These and other distinctions, important in much more recent research (see Bornstein 1987; Berlin and Kay 1991; Lyons 1995; Lewis 1997; Lindsey and Brown 2006; Caldiere, Jraisati and Chevallier 2008), lay well beyond Newton's particular scene of inquiry; they were not problematic for the theory he developed, and he did not address them extensively while remaining aware of individual, cultural and historical variability and conventionality of sensory perceptions.

Being ever mindful of the vagaries of language and his and others' difficulties in expressing new ideas; Newton "…explained [in the Latin *Optice*, 1706] that the designation of seven primary [that is, principal ?] colors is just a convention. Although we see a continual succession of innumerable colors in the spectrum, 'yet all of them may be comprehended under the species and names of the seven aforementioned [aforenamed ?] principal colors, even if their degrees may be innumerable' " (Newton, quoted in Shapiro 1994:626). Newton's point about the "species and names" of spectral colors was quite consistent with contemporary usages of those concepts by Robert Boyle [1627-1691], *The Skeptical Chymist* (1661), *Experiments and Considerations Touching Colours* (1664), *Origin of Forms and Qualities according to the Corpuscular Philosophy* (1666) and by John Locke [1632-1704], *An Essay Concerning Humane Understanding* (1690).

> "[Robert] Boyle's plain-speaking use of 'sort' and 'kinds' as synonyms for 'species' is especially noteworthy, for this usage was also [John] Locke's habit..." (Aarsleff 1982: n. 41, p. 77). Boyle wrote:
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> > "It was not at random that I spoke, when in the foregoing notes, about the origin of qualities, I intimated that it was very much by a kind of tacit agreement that men had distinguished the species of bodies, and that those distinctions were more arbitrary than we are wont to be aware of; for I confess that I have not yet ... met with any genuine and sufficient diagnostic and boundary for the discriminating and limiting the species of things; or to speak more plainly, I have not found that any naturalist has laid down a determinate number and sort of qualities or other attributes, which is sufficient and necessary to constitute all portions of matter endowed with them, distinct kinds of natural bodies: and therefore I observe that most commonly men look upon these as distinct species of bodies that have had the luck to have had distinct names found out for them, though perhaps divers of them differ much less from one another than other bodies, which (because they have been *huddled up under one name*) have been looked upon as but one sort of bodies." (Robert Boyle, *Origin and Forms of Qualities*, 1666, in *Works* III:49-50; emphasis added; quoted in Aarsleff 1982:56)

Aarsleff (1982) also cited many of John Locke's observations on the semiotic conventionality of the notion of "species and names" beginning with "... Locke's more general statement of the same principle [as Boyle's] in [*An Essay Concerning Humane Understanding,* 1690] III, ii,5: 'Because *Men* would not be thought to talk *barely* of their own Imaginations, but of Things as they really are; therefore they *often suppose their Words to stand also for the reality of Things*.'" And further:

> "The species of things are distinguished and made by chance in order to naming and names imposed on those things which either the conveniencys of life or common observation bring into discourse." (Locke, *Essay* draft, 1677)
>
> 'The minde or the man ... ranks [a certaine number of those simple Ideas] or else finds them ranked togeather by others under one general name, which we call a species or if more comprehensive a genus or in plaine English a sort or kinde." (Locke, *Essay* draft, 1677)
>
> "The *Essence* of each *Genus*, or Sort, comes to be nothing but that abstract *Idea*, which the General, or *Sortal* (if I may leave so to call it from *Sort*, as I do *General* from *Genus*,) Name stands for." (Locke, *Essay* III, iii, 15)
>
> "*Men make sorts of Things*. For it being different Essences alone, that make different Species, 'tis plain, that they who make those abstract Ideas, which are the nominal Essences, do thereby make the Species, or Sort." (Locke, *Essay* III, vi, 35)
>
> "'Tis Men, who taking occasion from the Qualities they find united in them, and wherein, they observe often several individuals to agree, *range them into Sorts, in order to their naming*, for the convenience of comprehensive signs. ... *The boundaries of the Species, whereby Men sort them, are made by Men*. ... So that we may truly say, such a manner of sorting of Things, is the Workmanship of Men." (Locke, *Essay* III, vi, 36, 37)

"…the Extent and Certainty of our Knowledge … has so near a Connexion with Words, that unless their force and manner of Signification were first well observed, there could be very little said clearly and pertinently concerning Knowledge." (Locke, *Essay* III, ix, 21)

Part of the contemporary confusions surrounding Newton's new theory of light and colors resided in the limitations of philosophical discourses, especially as to the "species and names" of colors as 'nominal essences made by men and mistakenly taken to stand for the reality of things.' That was a particular difficulty about "Whiteness"; the apparent color of sunlight, Newton demonstrated, was a heterogeneous compound, a confused aggregate promiscuously darting about, even while talk of it often *huddled up under one name*. Similarly implicated was Newton's notion that colors 'like in *species*,' that is, as nominal essences, could be 'engendered' by either singularly simple or multiply compound rays of light: "Colors may be produced by Composition which shall be like the Colors of homogeneal Light as to the Appearance of Colour, but not as to the immutability of Colour and Constitution of Light" (*Opticks* [1704]1979:132). Throughout the 17^{th} century the same critique that Boyle and Locke took up for *names* was taken up as well for *numbers* and graphical *diagrams* (all forms of signification). Numbers as 'signs of magnitudes' were not to be confounded with 'real essences.' Nevertheless, just such confusion is frequently encountered, as, for example, in some recent portrayals of Newton's ideas, where the 'innumerably many' colors in nature are *huddled up under one number*, the 'magical number seven' being the most prevalent (even pernicious) mis-characterization of the theory. And on that basis various interpreters have ascribed to Newton 'numerological beliefs' and commitments for which no textual evidence whatsoever exists.[31]

Naturalist philosophers, such as Boyle and Newton and Locke, were changing the landscape of scientific inquiry in fundamental ways. Earlier doctrines, which presumed a (pre-ordained) fixity, finitude, discreteness and fewness of the things of the world, had been bolstered by explanatory constructs invoking metaphysical/hypothetical notions such as 'substantial forms,' 'innate ideas,' and formal causes. Such notions preserved rather static understandings, but they fell short of accounting for many natural processes: whence arises the property of 'saltiness' in ordinary salt ($NaCl$), which is non-existent in the properties of sodium (Na) or chlorine (Cl); How can a perception of 'green' emerge from both a homogeneous stimulus of 'green-making' rays of light and a heterogeneous 'blending' of 'blue-making' and 'yellow-making' rays of light which have no 'greenness' inherent in them? What novel properties might emerge from combinations of chemicals or colors not yet tried? What contingent circumstances are involved in the generation of innumerably many colors or sounds?

"Traditionally, scientific explanations moved from the top—the abstract substantial form—down to the material particular: why x is f is explained by x partaking in F, where F is some form grounding f and conferring fs being onto objects by virtue of its own exemplification of F. The physicalization of forms, by emphasizing the role matter itself played in the generation of things, placed the explanatory priority on matter instead of forms. Thus the door to the bottom-up [sic] approaches of reductionism and emergentism were opened. …

"…in Descartes this [formal] causal relation implies that all ideas of secondary qualities [*qualia* such as pain, colors, tones] are innate ideas occasioned by these distinctive motions in the pineal gland (because he maintains causal likeness). But in Locke this causal relation led not to innatism but to a sort of emergentism as instituted by God. In Locke it is simply a law of Nature that when a brain's animal spirits are moving in a certain way, the mind connected to that brain has the idea of white. God can simply institute this causal relation because He is omnipotent. But the key notion here is emergentism.

Locke, and other corpuscularianism, were opening up a new kind of explanation. Reductionism and emergentism differ in what they are doing to their *explanadums*, of course, but there is a similar logic in how each is eliminating or segregating those properties. In both cases, some natural phenomena is associated with another, more metaphysically basic phenomena. These correlations are the explanatory engines behind both trends. When the idea of white (the emergent property) is correlated with certain motions in the brain's animal spirits (the basal properties), we take that to be a relative explanation for the idea of white. The difficult thing is the transition from a merely relative explanation to a (prima facie) absolute explanation. Now, I don't think that there is any 'reason' within the *explanans* itself that we can point to as justifying or securing that transition—and that I think is the key to the whole idea of emergentism! The transition lies not in the 'improvement' of the *explanans* such that it satisfies theoretical constraints like causal likeness and the Law of Non-being, but rather it lies in our rejection of those principles as necessary, or even desirable, for acceptable explanations. We begin to perceive the basic correlation itself as explanatorily significant, not as a sign for a stronger metaphysical relation but as constitutive of that relation itself" (Hill 2007:19, 18).

From the initial and multiplex characteristics of a physical 'musical string,' Newton's "Hypothesis" related a generative and parallel story, a plausible natural history, of how it comes to be that apparent colors or tones emerge (become present) in consciousness, how perceptibles become perceiveds. The story recounted a path from antecedent conditions to consequent effects. Acoustical and optical considerations, ran parallel, from physical circumstances through physiology, neurology, psychology, and lastly to the general shaping of cultural 'aesthetics' (music and painting) and the possibilities of 'harmony and discord' in any sensory domain. Newton closed his 'explanation of colors' with a canonical diagram and technical description of the orderly, systematic and measured relationships of the "more eminent species" of colors (those 'made by men') according to the appearance of the prismatic spectrum (a quasi-discretized continuum) to a sensitive and attentive observer. The description of spectral colors included an explicit 'tonometrical' aspect; like audition, human color perception, a concomitant of vision in general, entailed a determinable pattern of 'attunement' contingent upon corporeal being (that is, inherent to one's biological nature and operative in the domain of visual perception).[32] From there, Newton developed a more general and abstract COLORIMETER, describing and measuring geometrical character and behavior of color vision (one faculty of this 'curious harp of man's body'), its 'rule' (or measure and lawfulness) and its applicability to "All the Colours in the Universe which are made by Light". [33]

'A Revolution of all the Colours': Newton's Circular COLORIMETER

Many years after his original research in optics, and with the achievements of *Principia mathematica* (1687) accomplished, Newton turned toward revising his *Optical Lectures* (1671-2) for a new publication (the *Opticks* of 1704). A novel addition to his earlier work was a circular COLORIMETER. It generalized Newton's initial ideas about 'equilibrium' and 'equipollence' in mixing spectral colors, and it was presaged by his earlier tonometry (as well as, of course, Oughtred's circular slide rules).

"He [Newton] will spread himself in exhibiting the agreement of this [new, natural/experimental] philosophy with that of the Ancients and principally with that of Thales. … I also saw 'Three Books of Opticks': if it were printed it would rival the *Principia Mathematica*. … He pursues Catoptrics and Dioptrics … concisely and with elegance. He sets forth unheard-of wonders about colours. These he intends to publish within five years after retiring from the University. …" (mathematician David Gregory [1659-1708], 'Memoranda on Conversations with Newton', 5, 6, 7 May 1694; Newton, *Correspondence* III, No. 446, pp. 334-340).

Then, following some observations on Newton's ideas about *fluents* and *fluxions* (the calculus), computation of monetary interest, the orbital paths of comets, a liquid thermometer, and the motion of the moon around the earth, Gregory continued:

"A straight line drawn from the centre through the common center of gravity of circles proportional to the colours to be mixed and similarly situated in relation to them shows a circumference in the colour produced by the mixture.

The amount of the colours in pure (?white) light is as the harmonic divisions of a string" (David Gregory, 'Memorandum on Conversations with Newton', ? May 1694; Newton, *Correspondence* III, No. 448, pp. 344-348).

This was earliest announcement of Newton's circular COLORIMETER, which, as published a decade later in *Opticks* (1704), was designed to account for the perceptual regularities of mixed or compounded spectral colors, where the *qualities* and *quantities* of their specific ingredients were known: *"In a Mixture of Primary Colours, the Quantity and Quality of each being given, to know the Colour of the Compound"* (Newton, *Opticks*, 1704: Book I, Pt. II, Prop. VI, Prob. II; 1979:154-158). In function, this COLORIMETER was much like any other mathematical and mechanical instrument (paper or physical, imaginal or actual) such as clock dials, navigational compasses, thermometers, tonometers, or gauging rods enlisted as practical aids in measurements and calculations pertaining to natural phenomena. It coupled Newton's metrical geometry of the spectral dispersion (a canonical 'rule') to a 'law' of the forces or powers (supposedly) involved in generating the sensations and perceptions of color, a 'law' that was inspired, in part, by ideas about 'gravity' and the geometrization of forces developed before (and after) the *Principia mathematica* (see De Gandt 1995). Not only did this novel instrument provide a precise 'numerical measure' (quantification) for spectral colors and their perceptual mixtures, it eventually proved, some limitations aside, to be a generally correct and reasonably replicable account of human perception—that is, by "handling that Science [Optics] voluminously after a new manner, not only by teaching those things which tend to the perfection of Vision, but also by determining mathematically all kinds of Phænomena of Colours which could be produced by Refractions. For to do this, there is nothing else requisite than to find out the Separations of heterogeneous Rays, and their Mixtures and Proportions in every Mixture" (Newton, *Opticks* [1704]1979:131). In short, Newton's circular COLORIMETER was a specialized nomogram, a graphical expression of the "most regular, uniform, and constant Law" of prismatic refraction and a geometrical device for calculating the sensorial combinations of the innumerably many colors of light. What remains then is only to clarify the construction, calibration and operation of Newton's as presented in the *Opticks* of 1704, with special attention given to its 'musical' geometry and its implications pertaining to 'gravity.'

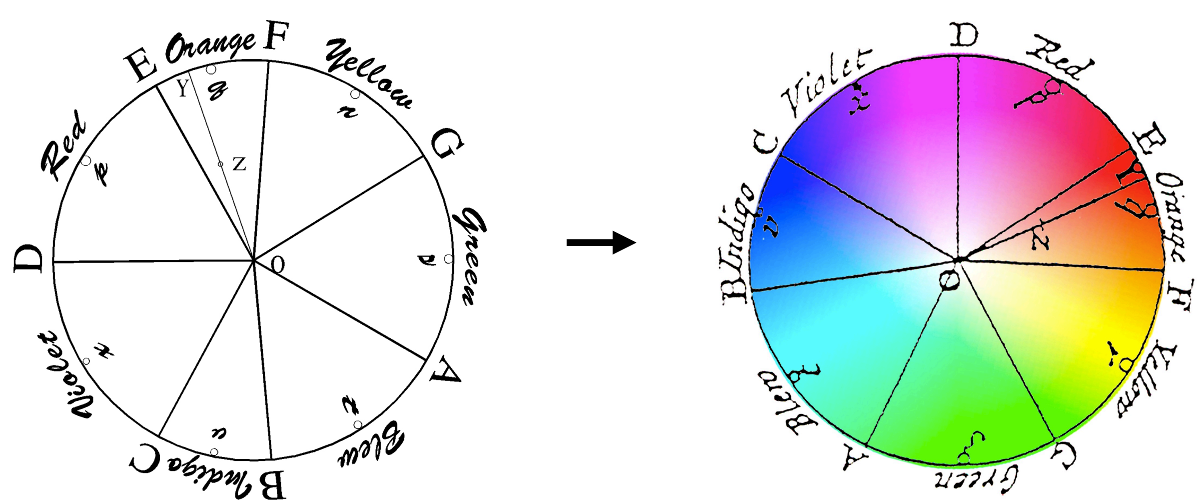


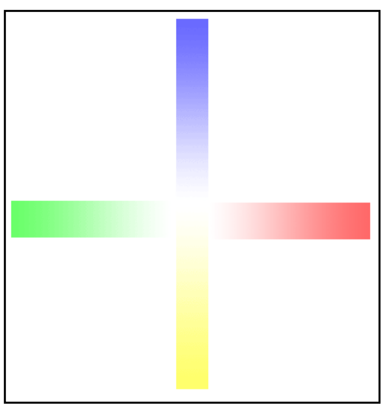
Figure 8
Newton's Color-Mixing 'Wheel with Many Spoaks'
(a. original (redrawn) → b. color enhanced and rotated)

> "With the Center O and Radius OD describe a Circle ADF [**Figure 8**[a]], and distinguish its Circumference into seven Parts DE, EF, FG, GA, AB, BC, CD, proportional to the seven Musical Tones or Intervals of the eight Sounds, *Sol, la, fa, sol, la, mi, fa, sol,* contained in an eight, that is proportional to the Number 1/9, 1/16, 1/10, 1/9, 1/10 [sic], 1/16, 1/9. Let the first Part DE represent a red Colour, the second EF orange, the third FG yellow, the fourth GA [sic] green, the fifth AB blue, the sixth BC indigo, and the seventh CD violet. And conceive that these are all the Colours of uncompounded Light gradually passing into one another, as they do when made by Prisms; the Circumference representing the whole Series of Colours from one end of the Sun's coloured Image to the other [**Figure 8**[b]], so that from D to E be all degrees of red, at E the mean Colour between red and orange, from E to F all degrees of orange, at F the mean between orange and yellow, from F to G all degrees of yellow, and so on" (Newton, *Opticks* [1704]1979:154-155).

The perimeter of the color wheel represents the continuous gradient of hues in the visible spectrum, extending from the least to the most prismatically refracted colors (**Figure 9**).

Figure 9: The Prismatic Spectrum, an *Image of the Sun*, a *Pillar of Colors*
"…a very pleasing divertissement…"
"a continual series…perpetually varying…without number"

The circle's center (***O***) indicates 'whiteness,' a 'mean' of all the prismatic colors. And any radius vector from the center to any point on the circumference of the circle represents a continuous gradient of the 'luminosity, intensity or fullness' (now called 'saturation') of some particular prismatic color (**Figure 10**):



"So, if to the Colour of any homogeneal Light, the Sun's white Light composed of all sorts of Rays be added, that Colour will not vanish or change its Species, but be diluted, and by adding more and more white, it will be diluted more and more perpetually." … … when several sorts of Rays are mixed, and in crossing pass through the same space, they do not act on one another so as to change each others colorific qualities, but by mixing their Actions in the Sensorium beget a Sensation differing from what either would do apart, that is a Sensation of a mean Colour between their proper Colours; and particularly when by the concourse and mixtures of all sorts of Rays, a white Colour is produced, the white is a mixture of all the Colours which the Rays would have apart, the Rays in that mixture do not lose or alter their several colorific qualities, but by all their various kinds of Actions mix'd in the Sensorium, beget a Sensation of a middling Colour between all their Colours, which is whiteness. For whiteness is a mean between all Colours, having it self indifferently to them all, so as with equal facility to be tinged with any of them" (Newton, *Opticks* [1704]1979:133; 159).

Figure 10: Saturation

While a prismatic spectrum displays a continuum of gradations, it also appears to sense to consist of several qualitatively different fields or regions, the several "more eminent species" of colors that are regularly ordered but of unequal extent. The musical-geometrical shape of the spectral image was defined originally by diligent inspection and measurement—the 'severalty' of the eminent colors being indexed to their 'severing' at their boundaries (as in **Figure 7**). Now, the circular plane of the COLORIMETER is also partitioned canonically, its 'sectors' coinciding with the rational tuning divisions (intervals or spaces) analyzed in Newton's original matho-musical exercises and enlisted throughout his optical researches. And so the structure of an audial system provided the fundamental calibration of the visual color-mixing wheel, echoing, if faintly, an ancient observation about the different functions of vision and audition:[34]

> *"The hearing reveals the visible entirely through interpretations,*
> *while sight announces the audible entirely through description."*
> (Claudius Ptolemy, *Harmonics* III.3 [c. 150 CE] 2000:94.1:141)

One cycle of the color circle comprises the extent of one 'musical octave' (**D-D'**, ratio **2:1**, or **720:360**), encompassing the range (*gamut*) of the whole series of visible spectral colors from one end of the spectrum to the other; and it is the sum of its partial divisions (that is, the ratios defining its constituent 'musical intervals'). The relative magnitudes of those ratios can be easily measured geometrically, in conventional ways such as 'by the clock' or 'by the compass' (with some help from "the wonderful business of logarithms"). An ordinary 12-hour clock dial is constructed by inscribing a regular dodecahedron in a circle, partitioning (sectioning or cutting) the whole circle into 12 equal-ratio or 'geometrically proportional' parts. Measuring the successive intervals of the whole cyclic 'octave' by 'clock units' (hours, minutes, seconds) is accomplished by calculation (**Equation 1**). There are but three different sizes of intervals (**T = 9:8**, **t = 10:9**, **S = 16:15**) arranged as **T S t T t S T** in the diatonic expression of Newton's spectral tuning. The central interval (**T**), **GA** in his circular diagram, defines the extent of the middle spectral 'greens', to which all else is "proportional with a refined symmetry" (Newton, c. 1671-2). Similarly, for the same sectioning or severing of the spectrum by the musical intervals the partial magnitudes can be measured by 'compass-units' yielding the central angular degrees of their respective arc segments (**Equation 2**).

$$\frac{12(\log\frac{2}{1})}{\log 2} = 12\,(\text{"hours"});\quad \frac{12(\log\sqrt[12]{2}/1)}{\log 2} = 1\,(\text{clock cycle} \equiv \text{octave})$$

$$\frac{12(\log\frac{16}{15})}{\log 2} = 1.117312853(\text{"hours"}),\ \text{or}\ 1\,\text{hr}.7\,\text{min}.2.325\,\text{sec}.$$

$$\frac{12(\log\frac{10}{9})}{\log 2} = 1.824037121(\text{"hours"}),\ \text{or}\ 1\,\text{hr}.49\,\text{min}.26.535\,\text{sec}.$$

$$\frac{12(\log\frac{9}{8})}{\log 2} = 2.039106017(\text{"hours"}),\ \text{or}\ 2\,\text{hrs}.2\,\text{min}.20.760\,\text{sec}.$$

Equation 1
Equal-12 Division ('clock')

$$\frac{360(\log\frac{2}{1})}{\log 2} = 360\,(\text{"degrees"});\quad \frac{360(\log\sqrt[360]{2}/1)}{\log 2} = 1\,(\text{circle} \equiv \text{octave})$$

$$\frac{360(\log\frac{16}{15})}{\log 2} = 33.52\,(\text{"degrees"})$$

$$\frac{360(\log\frac{10}{9})}{\log 2} = 54.72\,(\text{"degrees"});\quad \frac{360(\log\frac{9}{8})}{\log 2} = 61.17\,(\text{"degrees"})$$

Equation 2
Equal-360 Division ('compass')

So, the magnitude, say, of the interval **DE** (ratio **8:9**) is **2hrs. 2min. 20.760sec.** (by a 12-hour 'clock' cycle), and **61.17°** (by a 360-degree 'compass' cycle). These measurements are fully equivalent to Newton's specification — "describe a Circle and distinguish its Circumference into seven Parts DE, EF, FG, GA, AB, BC, CD, proportional to the Musical Intervals contained in an eight, that is proportional to the Number 1/9, 1/16, 1/10, 1/9, 1/10, 1/16, 1/9" — although undoubtedly, with his general mathematical acumen, he had little need to rely on such conceptual crutches as these. We are, however, free to use such descriptions, short of undermining an appreciation of Newton's preference for 'geometrical elegance.' But now the basic construction and calibration of the circular COLORIMETER can be summarized for the whole series of prismatic colors (**TABLE 4**), 'COLORS' referring here to "all degrees" of a "principall" or "more eminent species", for example all 'greens' between the most extreme 'yellowish green' (near **G**, the yellow/green boundary) and the most extreme 'bluish green' (near **A**, the 'grue/bleen' boundary), and so forth.

TABLE 4: COLORIMETER - Calibration

CALIBRATION — Isaac Newton's Color Wheel — 1 CYCLE=1 'OCTAVE'							
SECTOR	DE	EF	FG	GA	AB	BC	CD'
RATIOS	8:9	15:16	9:10	8:9	9:10	15:16	8:9
INTERVAL	T	S	t	T	t	S	T
COLORS							
ANGLES	61.17°	33.52°	54.72°	61.17°	54.72°	33.52°	61.17°

Newton then proceeded to give one example of the operation of the circular COLORIMETRIC ruler for the purpose of 'knowing the colors of mixtures,' the *quantity* and the *quality* of their constituents being given.

> "To give an instance of this Rule; suppose a Colour is compounded of these homogeneal colours, of violet [1] one part, of indigo [1] one part, of blue [2] two parts, of green [3] three parts, of yellow [5] five parts, of orange [6] six parts, and of red [10] ten parts. Proportional to these parts describe the [small] Circles *x, v, t, s, r, q, p,* [**Figure 8**] respectively, that is, so that if the Circle *x* be [1] one, the Circle *v* may be [1] one, the Circle *t* [2] two, the Circle *s* [3] three, and the Circles *r, q* and *p*, [5] five, [6] six and [10] ten. Then I find Z the common center of gravity of these Circles, and through Z drawing the Line OY, the Point Y falls upon the circumference between E and F, something nearer to E than to F, and thence I conclude, that the colour compounded of these Ingredients will be an orange, verging a little more to red than to yellow. Also I find that OZ is a little less than one half of OY, and thence I conclude, that this orange hath a little less than half the fulness or intenseness of an uncompounded orange; that is to say, that it is such an orange as may be made by mixing a homogeneal orange with a good white in the proportion of the Line OZ to the Line ZY, this proportion being not of the quantities of mixed orange and white Powders, but of the quantities of the Lights reflected from them" (Newton *Opticks* [1704]:1979:157).

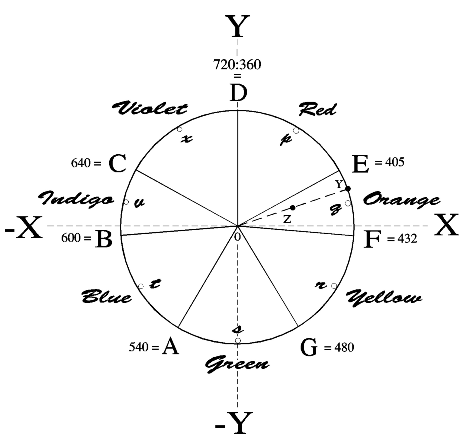


Figure 11
Newton's Color-Mixing Circle (redrawn)

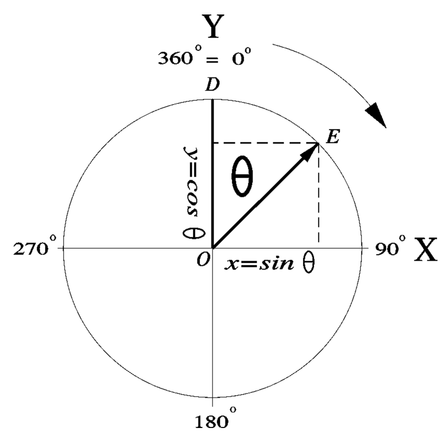


Figure 12
Conversions
Polar and Rectilinear Coordinates

To facilitate the discussion that follows, Newton's original color-mixing diagram is redrawn here (**Figure 11**). It shows the circle rotated and mapped to rectangular coordinates (±X, ±Y); and for convenience it also includes the string-length numbers from Newton's original system of musical tuning (manuscripts, c. 1665; **TABLE 1**) and its empirical confirmation in the orderly succession of the principal prismatic colors (**Figure 7**). Sepper (1994:206-207) has shown that the requirement — to distinguish the circle's circumference, proportional to parts, proportional to musical intervals, "proportional to the Number 1/9, 1/16, 1/10, 1/9, 1/10, 1/16, 1/9" — can be achieved by successive fractional reductions: a string-length 720 is first reduced by 1/9 (to 640), then that by 1/16 (to 600), that by 1/10 (to 540), that by 1/9 (to 480), that by 1/10 (to 432), that by 1/16 (to 405), and lastly that by 1/9 (to 360). The successive ratios of adjacent string-lengths are the ordered diatonic intervals, **T S t T t S T**, of Newton's musical system (**TABLES 1**, **2** and **3**) and of the 'prismatic tuning' (**TABLE 4**). Another diagram (**Figure 12**) supplies a reference orientation for the relation of rectilinear to polar coordinates.

The extent of the interval or space **DE** ("all degrees of red") is the *ratio* of the string lengths, **360:405 = 8:9** (= T), so that, in 'compass' terms **E**, which marks the boundary or "mean Colour between red and orange" in the spectral continuum, is precisely located **61.17°** right rotation from **D = 0**, at **1** radius distance from the circle's center O. That is, the location of E is expressible as an angular direction plus a radial distance, a vector (R, θ) or $\overrightarrow{OE}$ = **1, 61.17°** relative to the calibrated plane of the circular COLORIMETER. And so also for the rest of the intervals and colors according to the data in Newton's texts and diagrams. In the re-orientation of the COLORIMETER (**Figure 8**[b], **Figure 11**) the proportional symmetry of the spectrum to the 'mean in the green' (**AG**) is made more readily apparent as is the tonometric symmetry **T S t T t S T**. The eight boundary markers in the spectrum's *gamut* are 'alphabetically' centered on **D**, that is, **A B C <u>D</u> E F G**, to which the delimited intervals are rationally and symmetrically distributed (**CD=DE**, **AD=DG**, etc.). Moreover, while points **A** and **G** are the boundaries of the spectral 'greens,' they are 'by the numbers' **A = 540** and **G = 480**, respectively, the *arithmetic* and *harmonic* means of the tuning system. The divisions of the spectrum into the "more eminent species" of colors then corresponded to what Newton once called "the prime parts of an octave" that is, its first order 'diatonic' sectioning (in "Of Musick"), while also comprehending 'all the minutest gradations' of spectral colors.

Continuing with Newton's (now circular) metrical geometry, the specified parts of the COLORIMETER/TONOMETER (**DE, EF, FG**…) are arc or circular *segments*. In diagramming the linear spectral dispersion, the measuring line (a tensioned string or *kanōn*) was cut (sectioned, severed, divided, fractured) into constituent lengths at the observed boundaries of 'the more eminent colors' (in a manner reminiscent of Euclid's *Sectio canonis*). That fractioning was then analogized to the refrangibility of white light. It is now implied that the corresponding arc segments are delineated by a *secant line* intersecting (cutting) the circumference at two points, (e.g., **DE**, **EF**, **FG**…), the part of that line lying within the circular plane being the *chord* of the arc.

The 'small circles' (***p, q, r, s, t, v, x***) specified in Newton's colorimetric scheme indicate the 'centers of gravity' (centroids) of each of the successive arc segments (parts) of the spectrum, **DE**, **EF**, **FG**, etc. (again after Archimedes, *On the Equilibrium and Centers of Gravity of Planes*). Their significance is to expand and generalize the earlier principles of *equilibrium* and *equipollence* encountered in Newton's mixing of immediately adjacent colors (such as the *blue-green-yellow* example) in the linear spectral dispersion (**Figure 6**), based on the observed geometry whereby "in the middle of these parts each color appeared the most brilliant and intense of its own species". The new circular geometry made it possible to evaluate the mixing of *any* combinations of *any* spectral lights (homogeneal rays) within a planimetric *color space* (some placement in/of the *sensorium*), not just those of 'neighboring colors' in a linear distribution. All the colors in the spectrum are diagrammatically related to white light as a circumference is to a center of a circle. In exact geometrical terms those partial 'centers of gravity' would not be located directly on the circle's circumference, and Newton did not illustrate them there, but to simplify the computations below they will treated here as simple 'compass' points of each arc's bisection.[35]

In the circular COLORIMETER, Newton's late move to characterize perceptual color-mixing echoes some of the developed concepts of the *Principia mathematica*, albeit rather informally. A spatially extended physical body (possessing both volume and density), such as the moon, bears a gravitational relationship of mutually attractive forces to other such bodies, such as the Earth, the forces of varying according to the centers of their mutual distances and masses *as if they were point masses*. Thus, for example, in the color circle the small circle ***t***, the geometrical center or mean of 'all degrees of blue' contained within the boundaries of the extremes **AB**, is interpretable as an equilibrium point and location signifying both itself singularly (a particular 'blue') and collectively 'all the blues.' And so for the other small centers of 'gravity.' This notion reduces the calculation problem (for the purpose of illustrating 'an instance of the rule') to finding 'the common center of gravity' of a combination of specified "primary colors" (while not excluding the possibilities of mixing *any* spectral colors whatsoever). The centers of gravity indicate, in fact, "primary" and "homogeneal" colors in Newton's usual sense of being "least parts" of light effected by refraction (rather than the grosser and variable categories of the "more eminent species") as required by the stated problem: *"In a Mixture of Primary Colours …"*. The blending of those particular ingredients is regarded as part of a *system* of interacting constituents. And, since the colors in the spectrum are harmonically (that is, canonically) apportioned, a relationship between 'harmony' and 'gravity' is implied.[36]

For the sample color-mixing problem, Newton supplied a quantification (or 'weighting') of the small 'centers of gravity' in the simple terms of proportional parts, much like in mixing colored pigments one could say of a compound consisting of **10** parts, that **3** are 'blue' and **7** are 'yellow,' the resulting 'alloy' being a markedly 'yellowish green,' a green inclining more to yellow that to blue (to paraphrase Newton). Newton's 'quantities' were characteristically rational (as 'ratios of quantities to other quantities, usually those of the same kind') rather than absolute measures, whether of string-lengths and tones, colors, geometrical areas, or the 'units' in mathematics or celestial dynamics:

> "In the centuries following the publication of the *Principia*, the fundamental units of classical dynamics have been mass, length, and time, associated with dimensional analysis. … In the *Principia*, however, Newton is generally not concerned with units or dimensionality. The reason is that, except for such quantities as distances fallen as measures of gravity, lengths of seconds pendulums at various places, and the differences in size of the earth's equatorial and polar axes, Newton tends to be concerned with ratios of quantities rather than with quantities themselves. Thus he does not establish a unit of mass and compute individual masses. … although he has a unit of force [pound weight] … he does not compute the magnitude of specific gravitational or dynamical forces. Rather, he compares masses with other masses, and forces with other forces" (I. Bernard Cohen, "Guide," to Newton, *Principia mathematica* [1687]1999:92).

In perceptual color-mixing, Newton now claims, the various *actions* or *forces* of different monochromatic prismatic lights combine *within the sensorium* to engender new color qualities. The sample problem presented for measuring the mixing of prismatic colors is framed in the language of partial color 'forces' and the combinations or sums of their 'actions,' their "powers and dispositions to stir up sensations." The existence and behaviors of such 'forces' or 'powers' was adduced from the effects of perception and geometrical measurements, "in Experiments, where Sense is Judge," as "something or other" acting on and within the *sensorium*.[37]

DEFINITION

> "The homogeneal Light and Rays which appear red, or rather make Objects appear so, I call Rubrifick or Red-making; those which make Objects appear yellow, green, blue, and violet, I call Yellow-making, Green-

making, Blue-making, Violet-making, and so of the rest. And if at any time I speak of Light and Rays as coloured or endued with Colours, I would be understood to speak not philosophically and properly, but grossly, and accordingly to such Conceptions as vulgar People in seeing all these Experiments would be apt to frame. For the Rays to speak properly are not coloured. In them there is nothing else than a certain Power and Disposition to stir up a Sensation of this or that Colour. For as Sound in a Bell or Musical String, or other sounding Body, is nothing but a trembling Motion, and in the Air nothing but that Motion propagated from the Object, and in the Sensorium 'tis a Sense of that Motion under the Form of Sound; so Colours in the Object are nothing but a Disposition to reflect this or that sort of Rays more copiously than the rest; in the Rays they are nothing but their Dispositions to propagate this or that Motion into the Sensorium, and in the Sensorium they are Sensations of those Motions under the Forms of Colours" (Newton, *Opticks* [1704]1979:124-125).

Two aspects of these 'powers' or 'actions' are given. The calibration of the COLORIMETER provided exact indications of prismatic color *quality* (in appearance to an observer), that is, of homogeneal rays as points on the circumference in the spectral continuum. The *quantity* of colors in the sample mixture was supplied as a proportional 'number of parts' [10 of 'red' (*p*),' 1 of' blue' (*t*), etc.], that is, as *relative* quantities within the whole compound (totaling 28 parts).

For Newton's color-mixing wheel the precise locations of the eight sounds ('tones'), D, E, F, G, A, B, C, the magnitudes of the intervals (spaces, segments) they delimit, T S t T t S T, as well as the 'centers of gravity' (*p, q, r, s, t, v, x*) of those parts (where the sample colors appear "most intense and florid" of their kind), are given in TABLE 5 by both polar and rectangular coordinates on the planimetric diagram.[38] These locations uniquely index perceived prismatic color *qualities* along the spectral continuum (with as many intermediate gradations as one might please to notice) while the proportional *quantities* Newton provided for the sample mixture are also listed.

TABLE 5: 'An Instance of the Rule'

TONE VECTORS	COLOR VECTORS		QUANTA	(R, _) R = 1	COORDINATES X [=sin _]	Y [=cos _]
D (*sol*)				(1, 0.00°)	0.000000000	1.000000000
	p		10	(1, 30.59°)	0.508891180	0.860830858
E (*fa*)				(1, 61.17°)	0.876054314	0.482212441
	q		6	(1, 77.93°)	0.977892858	0.209106568
F (*mi*)				(1, 94.69°)	0.996651672	–0.081764561
	r		5	(1, 122.05°)	0.847585331	–0.530659123
G (*la*)				(1, 149.41°)	0.508891180	–0.860830858
	s		3	(1, 180.00°)	0.000000000	–1.000000000
A (*sol*)				(1, 210.59°)	–0.508891180	–0.860830858
	t		2	(1, 237.95°)	–0.847585331	–0.530659123
B (*fa*)				(1, 265.31°)	–0.996651672	–0.081764561
	v		1	(1, 282.07°)	–0.977892858	0.209106568
C (*la*)				(1, 298.83°)	–0.876054314	0.482212441
	x		1	(1, 329.41°)	–0.508891180	0.860830858
D (*sol*)				(1, 360.00°)	0.000000000	1.000000000

Newton expressed the problem of 'knowing' the color of a mixture of prismatic colors as that of finding (calculating) the 'common center of gravity' (*Z*) of its constituents, a set of partial 'centers of gravity' (p, q, r, s, t, v, x), "the Quantity and Quality of each being given". What remains is "*to find Z the common center of gravity*" (Equation 3) as an indicator of the compound color resulting from the mixture of the given constituent colors (ingredients), that is, from the sum of their 'actions' as partial color 'forces,' their "Power and Disposition to stir up a Sensation of this or that Colour".[39] Those colorific (color-perception-making) 'forces' are designated here as $f_C = m_C v_C$, that is, as the product of a prismatic color's *quantity* or *'mass'* (m_C), its relative proportion within the whole compound, and its *quality* (v_C), its unique 'hue' in the spectral continuum as indicated by a vector direction. The resultant (or compounded) color is a *sum* of those 'partial color forces' (as prismatic lights are added to lights). Hence, the *power* or *disposition* of a prismatic color to stir up a sensation is measured (determined) as a force vector, indexed by its relative *position* (polar or rectilinear) within the spectral continuum *and* its relative *quantity* (as assigned) within the spatial design of the COLORIMETER.

$$\boxed{Z\ (common\ center\ of\ gravity) = \frac{m_1 v_1 + \ldots + m_k v_k}{m_1 + \ldots + m_k}}$$

Equation 3
Z (common center of gravity)

In order to calculate Z and to determine its location within the plane of the circular COLORIMETER, the rectangular coordinates (±X, ±Y) of the individual hues (where R=1) are proportionally quantified (or 'weighted') and the results summed (TABLE 6):

$$(Z_{x,y}) = 10/28p_{x,y} + 6/28q_{x,y} + 5/28r_{x,y} + 3/28s_{x,y} + 2/28t_{x,y} + 1/28v_{x,y} + 1/28x_{x,y}$$

For example, from TABLE 4 above, the rectangular coordinates for r (the central 'yellow' color located on the most extreme circumference of the color-mixing circle) are X = 0.847585331 and Y = –0.530659123. Multiplying each of those by the given *relative* quantity of that 'yellow' in the mixture (that is, 5 of 28 total parts), so that X = 0.151354523 and Y = –0.094760557, proportionally reduces the relative 'strength' of that color's 'force' (its particular 'yellow-making' contribution) in the resultant compound without changing its specific quality (hue) as indicated by its vector direction. And so for the others.

TABLE 6: The Geometry of 'Partial Color Forces'

MIXTURE PROPORTIONS		QUANTIFIED COORDINATES X	Y
10/28 *p* =		0.181746850	0.307439592
6/28 *q* =		0.209548469	0.044808550
5/28 *r* =		0.151354523	–0.094760557
3/28 *s* =		0.000000000	–0.107142857
2/28 *t* =		–0.060541809	–0.037904223
1/28 *v* =		–0.034924744	0.007468091
1/28 *x* =		–0.018174685	0.030743959
28/28 'parts'		Z_X = 0.428996615	Z_Y = 0.150652555

The summed rectangular coordinates ($Z_{X,Y}$) give the precise location of Z, ***the common center of gravity,*** within the color-mixing circle, indicating the specific color compounded from the **28** given ingredients. These coordinates can be converted (by any of the formulas in **Equation 4**) to the polar direction of Z from the center (O) and through to its extension at Y on the circumference. The result is precisely as Newton stated and diagrammed (see **Figures 7, 10**): *"from the center [O] … through Z to the Circumference, drawing … OY," so that "the Place of the Point Y in the Circumference shall shew the color* [quality] *arising from the Composition of all the Colours in the given Mixture."*

$$\boxed{\theta_Z = \tan^{-1}\left(\frac{Z_X}{Z_Y}\right);\ \theta_Z = \cos^{-1}\left(\frac{Z_Y}{\sqrt{Z_X^{\,2}+Z_Y^{\,2}}}\right);\ \theta_Z = \sin^{-1}\left(\frac{Z_X}{\sqrt{Z_X^{\,2}+Z_Y^{\,2}}}\right)}$$

Equation 4
Conversions - Rectangular to Polar Coordinates

Consequently, the vector angle (θ_z) of the compounded color ($\overrightarrow{OZY}$), a directional indicator of the resultant color quality (its specific hue) in the spectral continuum, is found to be **70.65°** 'clockwise' from **D** [= **0°**]. It intersects the circumference of the mixing wheel at the point Y amidst the various 'degrees of orange' (between **E** and **F**) in the spectral continuum or, again in Newton's own phrase: *"the Point Y* [at **70.65°**] *falls upon the circumference between E and F, something nearer to E* [at **61.17°**] *than to F* [at **94.69°**], [therefore] *… the colour compounded of these* [**28**] *Ingredients will be an orange* [at **70.65°**], *verging a little more to red than to yellow."* *Qualitatively* this specific homogeneal 'orange,' lying along the radius vector $\overrightarrow{OZY}$, is slightly more 'reddish' than the 'medial' orange q (at **77.93°**). Moreover, it is measurably determined and enjoys an exact metrical relationship with all other spectral colors. For **R = 1**, the rectangular coordinates of the point Y on the circumference of the color-mixing circle are **X = .943512164, Y = .31337888**.

Along the vector $\overrightarrow{OZY}$ the point Z in the circular diagram is located between the center (O), indicating total 'whiteness,' and the circumference (Y), indicating the completely full or saturated spectral color quality (the singularly homogeneal and particular slightly reddish 'orange' which matches the compounded color). Therefore, the distance OZ—which indicates the *quantitative* property of the compound color Z (*"proportional to the Fulness or Intenseness of the Colour, that is to its distance from Whiteness"*)—along $\overrightarrow{OZY}$ can also be determined (where OY is now known to be **R,** θ **= 1, 70.65°**). The magnitude of OZ is a proportion of the magnitude of OY such that, by the rectangular coordinates of Z given above its relative value is calculable: OZ/OY (**Equation 5**).

$$\boxed{OZ = \frac{\sqrt{Z_X^{\,2}+Z_Y^{\,2}}}{OY} = \frac{0.454680423}{1}}$$

Equation 5
OZ/OY : Proportion of Color Intensity (Saturation)

And thus, again in Newton's own phrase: the magnitude *"OZ is a little less than one half* [about **45.5%**] *of OY, and … this orange hath a little less than half the fulness or intenseness of an uncompounded orange; that is to say, that it is such an orange* [the specific color indicated at Y =

70.65°] *as may be made by mixing an* [that very one] *homogeneal orange with a good white in the proportion of the Line OZ to the Line ZY"* [that is, in a ratio, approximately, of **.455 to .545** or **9 to 11**]*, this Proportion being not the quantities of mixed orange and white powders, but of the quantities of the* [predominate] *Lights reflected from them."*

According to Newton's 'rule' of prismatic color 'forces' (or 'powers') and their various "Actions mix'd in the Sensorium", calculations confirm precisely the colorific *quality* ('hue') and *quantity* (relative 'intensity' or saturation) of a sample color (*Z*) compounded from the lights of several "homogeneal" (prismatic) colors, "the Quantity and Quality of each being given". The circular COLORIMETER was calibrated as a TONOMETER (inspired by Oughtred's *Circles of Proportion*), that is, by the 'metrical geometry' of a tuning system that defined the proportional distribution of the several "principall" or "more eminent species" of colors (where their distances and separations had been, originally, 'diligently examined' and measured in linear canonical terms). The 'centers of gravity' indicated a "primary" and "homogeneal" color at a position where "each color appeared the most brilliant and intense of its own species". The resultant location of the color Z, compounded from given ingredients, that is, from their various 'color forces,' was to be found by 'reading it off' on the calibrated COLORIMETER. The operation is confirmed by calculation, that is, Z was determined to be the 'center of gravity' common to several partial 'centers of gravity' (p, q, r, s, t, v, x) and its exact position relative to the center (O) within the circular plane was given by rectangular coordinates as $\sqrt{(Z_X^2+Z_Y^2)}$; and, by polar coordinates the vector $\overrightarrow{OZ}$ was found to be **.455 R, 70.65°**, indicating a particular saturation of a particular color of a 'reddish' orange.

Newton claimed a universal validity for his colorimetric 'rule' of the normal perception of light and colors and the 'law of forces' governing their various mixtures in the sensorium. It was constructed and verified upon the accumulated evidence of very considerable experimentation and demonstration in optical research, a 'commerce with sensible nature' "in Experiments, where Sense is Judge", coupled with a sophisticated mathematical intuition. It was consistent with contemporary notions about the 'perceptual acquaintance' with natural phenomena and the prevailing idea that all the senses operated in much the same way, by the detection, ratiocination and interpretation of 'motions.' The late *Opticks* continued the general objectives of the *Principia mathematica* (though not realizing them as fully), to clarify "the laws and conditions of motions and of forces for the fundamental topics of … light and sounds". Moreover, the COLORIMETER was, for Newton, not only imbued with the cachet of ancient wisdom (*via* its noble canonical construction and echoes of the long-standing 'harmonic' principles of system and symmetry) but it promised to be a practical instrument in entirely modern contexts.

> "This Rule I conceive accurate enough for practice, …; and the truth of it may be sufficiently proved to Sense. …"
>
> "All the Colours in the Universe which are made by Light, … are either the Colours of homogeneal Lights, or compounded of these, according to the Rule …"
>
> "… Thus it is by the computation: and they that please to view the Colours made by a Prism will find it also in Nature" (Newton, *Opticks*, [1704]1979:158, 164).

It was not long before Newton's 'rule' was put to test. "As well as his 'musical compasses' and his work on the dynamics of the vibrating string, [Brooke Taylor—musician, painter, mathematician] collated substantial material for a treatise on musical ratio theory; and in his 1719 book on Perspective [*New Principles of Linear Perspective*] he took up Newton's idea of a 'colour wheel', reproducing a similar diagram with a somewhat expanded explanation" (Wardhaugh

2008:182). Pertinently, Taylor evaluated Newton's casual claim that that COLORIMETER could be a practical instrument for predicting the predominate hues arising from the mixing of colored *substances* as well as those of prismatic *lights*.

> "In an appendix to a new edition of his treatise on linear perspective Taylor tried to apply Newton's mixing diagram, and in doing so discovered not only that light colors overcome dark but also that the products of pigment mixture were quite unpredictable:
>
> > 'If the nature of the material Colours, which are used in Painting was so perfectly known, as that one could tell exactly what Species of Colour, how perfect and what degrees of light and shade each material has with its respect to its Quantity, by these rules one might exactly produce any Colour proposed, by mixing the several materials in the just proportions. But … these Particulars cannot be known to sufficient exactness for this purpose, besides the Tediousness that would be in Practice. … But these Properties of particular Materials I leave to be considered by the Practitioners in this Art' " (Brook Taylor, quoted in Gage 1993:169).

Analogies are greatly important in scientific investigations and Newton's 'double talk' about the relationships of sonorific and colorific qualities was paradigmatic of the 17th century preoccupation with the 'consents and dissents of audibles and visibles.' But analogies, which are certainly useful fictions, become most interesting scientifically at the points they cease to hold true; it is not their successes but their limits and failures that propel inquiry forward. For all of Newton's attempts at precision and fusion, the subjects and nature of acoustics and optics, audition and vision, sounds and lights, colors and tones, painting and music, are complex and vastly different. And Newton enjoyed very little in the way of study, practical experience, or sustained interest in the *arts* of music and painting, personal limitations he never sought to hide. Once he had completed the original musical-geometrical description of the prismatic spectrum (as in **Figure 7**), he conjectured that it "not only … agrees with the [visible] phenomena very well, but also … perhaps involves something about the harmonies of colors (such as painters are not altogether unacquainted with, but which I myself have not yet sufficiently studied) perhaps analogous to the concordances of sounds" (Newton, *Optical Papers*, *Optica* II*, Lecture* 11*,* c. 1671-72).[40] In 1677, John North [1645-1683], master of Trinity College Cambridge and a friend of Newton and Barrow, asked Newton to review "A Philosophical Essay of Musick" by Francis North. Newton invoked a wave/pulse model of the propagation of sound to clarify the perception of 'concords',[41] but he begged off delving further into musicological issues: "…there being some things which I cannot speak positively to for want of experiments and skill in Musick. … I want experience & skill to enable me sufficiently to judge what follows [in the essay] about Tunes, ye scale of Music, & consort; this requiring a combination of musical & Mathematical skill" (Newton, *Correspondence* II:205, 207).

Some of Newton's analogies (there are many) between light and sound, tones and colors, are plausible enough in very limited ways. For example, 'color is the pitch of light,' it has been said, so as the perception of pitch is proportional to audible frequency the perception of color is proportional to visible frequency. These relationships were asserted in the '*Queries*' to the *Opticks* of 1704.[42] There are also *qualitative* phenomenological characteristics in the perceptions of musical *intervals* (*ratios* of pitches); these are what Isaac Barrow (and others) referred to as an interval's *determinate habitude* ("… from the Length of two Strings differing according to a certain Proportion, both stretched alike, if both be struck with a Quill, there is found a determinate Habitude of the Sounds produced"). The characteristic qualities in the perceptions of an interval, say, of 'third *major*' (frequency ratio **5:4**) and 'third *minor*' (ratio **6:5**), which are notoriously difficult to 'name,' were conventionally distinguished and called simply *major* and *minor*. This 'determinate habitude' of musical intervals makes it possible for a melody (a particular succession of intervals) to be recognized as the 'same tune' when sung in different pitch registers, by bass or soprano, or, alternately, when played on a violin or a piano.

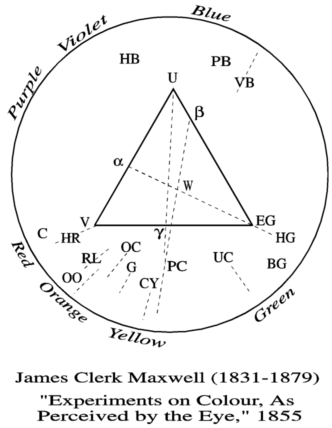
As for what Newton referred to as the 'name and nature' of such qualities, Galileo had put the matter clearly (although not entirely through to the sensorium):

> "… the length of musical strings is not the direct and immediate reason behind the forms [perceptible qualities] of musical intervals, nor is their tension, nor their thickness, but rather the ratio of the numbers of vibrations and impacts of air waves which go to strike our ear drum, which likewise vibrates according to the same measure of times" (Galileo Galilei, *Two New Sciences* [1638]2000:149-150).

Galileo went on to characterize the perceived quality of an interval of the 'fifth' (ratio **3:2**) comprised of 'two vibrations whose times are in the ratio of three to two:' [it] "… produces a tickling and teasing of the cartilage of the ear drum so that the sweetness is tempered by a sprinkling of sharpness, giving the impression of being simultaneously sweetly kissed, and bitten." So, in musical matters, to the *same* sonorous *ratio* of frequencies belongs the *same* perceptible quality, but Newton's canonical partitioning of the prismatic spectrum of colors (see **Figures 7, 11; TABLE 4**) resulted in just the opposite observation about the *ratios* in the spectral colors, that is, to the *same* ratio in the colorific spectral dispersion belong *different* perceptible qualities. As noted above, in Newton's canonically tuned spectrum the spaces (intervals) of *Red, Green* and *Violet* are all ratio **9:8**, and the spaces occupied by *Blue* and *Yellow* measure the same (ratio **10:9**), as do those for *Indigo* and *Orange* (ratio **16:15**). But the phenomenal (manifest) qualities of the colors indexed to these 'equal intervals' are decidedly different; 'green' is certainly not 'red,' nor is 'blue' the same as 'yellow,' either nominally or perceptually. Consequently, on the evidence, another '*Query*' of the *Opticks* is far more problematic than the previous ones.[43]

There were many compelling reasons for Newton to adopt and sustain a 'musical tuning of the prismatic spectrum.' An appeal to *canonic*s, the 'severing' of a tensioned string (in both its older 'musical' sense using integer ratios and more modern 'geometrical' one of logarithmic divisions), served Newton well indeed to help understand the prismatic spectrum's continuum of color gradations, at many 'levels of scale' from the minute and innumerably many to some several "more eminent species". It facilitated his abstract thinking about system and symmetry, as well as concrete considerations of the separations and compoundings of spectral lights (whose measured distributions in the "Image of the Sun" were found to be "in proportion as the Differences of the Lengths of a Monochord which sound the Tones in an Eight, *sol*, *la*, *fa*, *sol*, *la*, *mi*, *fa*, *sol*." Applications of canonical methods to phenomena, however, like many other invocations of metrical geometries (as in clocks or thermometers), are hardly determinative of cultural acoustical/optical aesthetics.[44]

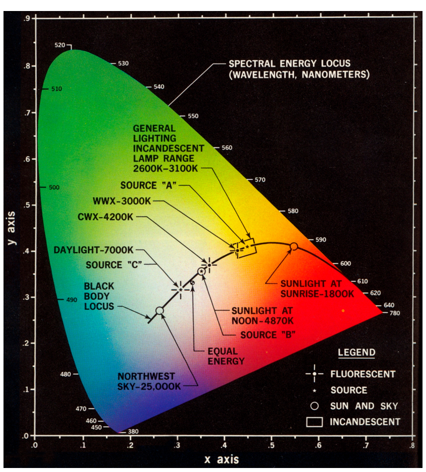
Despite the shortcomings of Newton's analogical/metaphorical elaborations of a theory of light and colors, historical developments in colorimetry sustained and reinforced much of his confidence in the legacy of Willliam Oughtred and the value of systematic measurement, even while eventually abandoning his particularly 'musical' techniques and speculations about 'harmony.' For a while, 'musical' analogies continued to inform research into vision and the optics of light and colors. In the early 19th century Thomas Young [1773-1829] employed them in the development of the trichromatic theory of the 'attunement' of visual receptors (see Young 2002; Pesic 2014). But after about the mid-19th century many such analogies were no longer thought to be scientifically productive—for example, in Helmholtz's works on physiological optics [1866]1962 and physiological acoustics [1877]1954—the neuro-physiological-cognitive *differences* of vision and audition coming more to the fore than their similarities. Newton's core idea of some *general* *rules/laws*, accounting for the engendered geometry of perceptual color space, endured nevertheless, and it came to be confirmed in large by James Clerk Maxwell [1831-1879]. Using three 'chromaticity coordinates' Maxwell mapped perceptual color space by 'triangulating' the locations of particular colors and their mixtures (**Figure 13**).[45]



> "The rule for the composition of colours is identical with that for the composition of forces in mechanics." (James Clerk Maxwell)

Figure 13: Maxwell's 'Chromatic Teetotum'

During the early 20th century, rigorous psychophysical measurements established that human vision was normally capable of differentiating some one million colors. Systematically matching up the 'physical' and 'psychical' parameters of light and colors, the *Commission Internationale d'Eclairage* (C.I.E., 1931) mapped the technical data into standardized *color space diagrams* (**Figure 14a**).



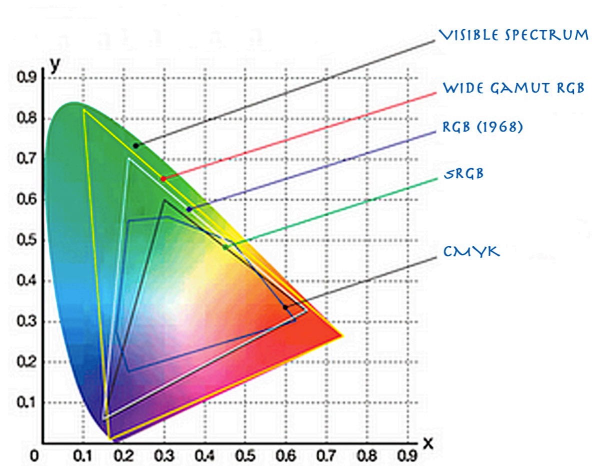
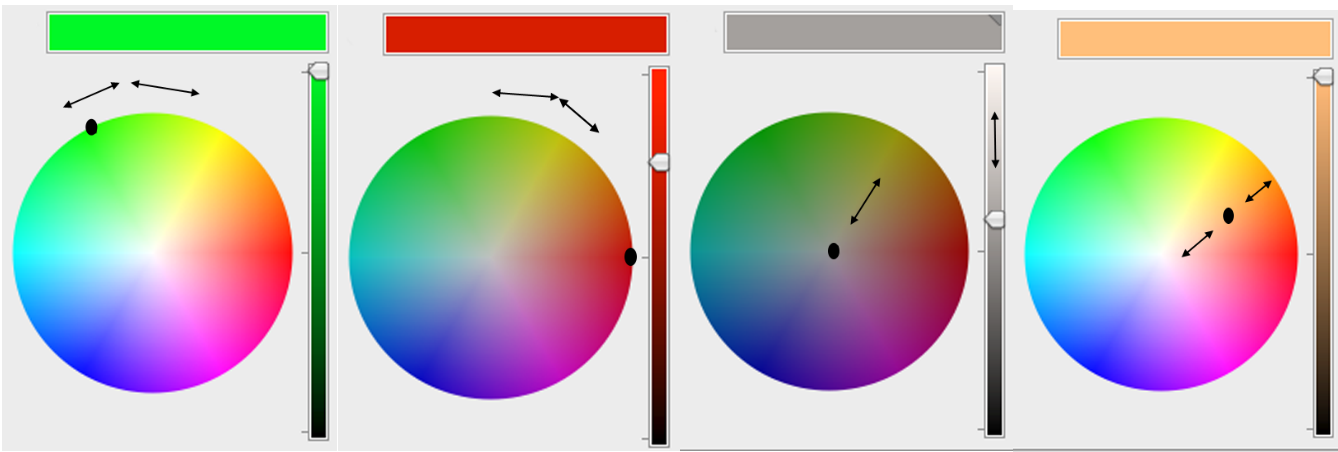


Figure 14: C.I.E Diagrams

a. locus of visible spectral energy b. *gamuts*

The C.I.E. diagrams are the most advanced echoes of Newton's color mixing circle, with the color mean to them all, the white light of the sun, situated at the center of the spectral energy locus. Generally in those diagrams no particular divisions or categorizations of the spectrum into a succession of some "more eminent species" of colors are definitely indicated. And nothing of Newton's original 'canonical metrics' survives overtly, save perhaps in the modern references to the *ranges* of colors that can now be generated and matched to sense by artificial technological means (e.g., the systems RGB, CMYK, NTSC, etc.) as *gamuts* (**Figure 14b**; "All the notes in the Gam ut may be tuned…"). Moreover, to this day, Newton's circular colorimetric rule continues to be "sufficiently proved to Sense" and "accurate enough for accurate practice". Circular color selectors with continuous interactive controls, their metrics embedded in programming for hue (circumference), saturation (radius) and lightness are now commonplace (**Figure 15a-d**).



a b c d

Figure 15: Interactive Color Selection (RGB Gamut)

a. a middle green at full saturation and full lightness **b.** a middle red at full saturation and 75% lightness **c.** a 'white' (all colors) at 60% lightness **d.** an orange verging a little more to yellow than to red, at 55% saturation and full lightness

* * * * * * * * * * * * * * * *

NOTES

[1] For an overview see: Crombie [1964]1990, [1990]1996; Yolton 1984; Erlmann 2010; Gal and Chen-Morris 2010; Buchwald and Feingold 2013.

[2] Newton's interests and investigations into the nature of the senses and perceptions contradict much of the 'usual story' about modern science. For example, Koyré (1965), assessing "The Significance of the Newtonian Synthesis," summarized a predominate version of that mythos thus:

> "… we have all, or nearly all, accepted the idea of the Newtonian world machine as the expression of the true picture of the universe and the embodiment of scientific truth … for more than two hundred years such has been the common creed, the *communis opinio*, of modern science and of enlightened mankind. … It is possible that the deepest meaning and aim of Newtonianism, or rather, of the whole scientific revolution of the seventeenth century, of which Newton is the heir and the highest expression, is just to abolish … the world of sense perception, the world of appreciation of our daily life, and to replace it by the (Archimedean) universe of precision, of exact measures, or strict determination. … Modern science [substituted] for our world of quality and sense perception, the world in which we live, and love, and die, another world—the world of quantity, of reified geometry, a world in which, though there is a place for everything, there is no place for man."

But alas, in that respect, Isaac Newton was neither a 'Newtonian' nor a proponent of 'Newtonianism.' For him, the human phenomena of sensorial appearances and perceptual acquaintances (what he called 'manifest qualities'), engendered in auditory and visual experiences (that is, in the very acts of seeing and hearing, among others) were fully essential to philosophical inquiry, not to mention life in general.

In student notebooks (*Certain Philosophical Questions* [1664-5], McGuire and Tamny [1983]2002) Newton surveyed the consensus of the times:

> "*On Sensation* … The common sensorium is either: (1) the whole body, (2) the orifice of the stomach, (3) the heart, (4) the brain, (5) the membranes, (6) the *septum lucidum* [a membrane separating the cerebral hemispheres], (7) some very small and perfectly solid particle in the body, (8) the conarion [the pineal body, strictly speaking the 'seat' of common sense according to Descartes: *Of the Senses in General*, "It is the mind that senses and it is situated in the brain, where it exercises that faculty which is called common sense"], (9) the concourse of nerves about the 4th ventricle of the brain, or (1) the animal spirits in that 4th ventricle."

The activities of perception and sensation ('sensing') were understood to take place in what was called the *sensorium*, in much the same way that 'hearing' is what properly occurs in an *auditorium* or 'swimming' in a *natatorium*. Particular 'perceiving' and 'sensing' (audition and vision *intra alia*) were situated within the bodies of individuals, particular and vital perceiving beings, and that embodied condition rendered them sometimes problematic relative to natural philosophy's quest for universal truths (Frank 1990; Rousseau 1990, 2004; de Pater 2005; Wolfe and Gal 2010; Buchwald and Feingold 2013). Undaunted and undeterred by 'limitations' however, Newton went on to surmise that significant principles of 'sensing' and 'perceiving' derived from natural biological functioning as well as a participation in the circumstance of universal being: *"spatium universuum, sensorium est entis incorporei, viventis, et intelligentis?"* [Is not universal space the sensorium of a being incorporeal, living and intelligent?] (Newton, *Optice*, 1706, Query 20; also see *Opticks* (after 1717), Query 28, the late 'General Scholium' of *Principia mathematica*).

[3] Throughout this essay I have adopted a practice of quoting Newton in rubricated text, in the manner used to advantage in Stillman Drake's 'dialogue' on Galileo Galilei's first book on physics, and for much the same reasons: "Galileo's [Newton's] exact words. Many notions are still ascribed to Galileo [Newton] for which direct evidence cannot be found in his precise words but which depend upon preconceptions left unexamined by those who assert them. The acuteness of Galileo's [Newton's] refutations of opponents leave little doubt that he examined his own assertions with equal care and that they conform to some standpoint from which they appeared to him consistent and in agreement with his observations. It has become popular to impute internal contradictions to him and to account for those by supposing him to hold mystical views and an unreasoning zeal for certain ideas, …" (Drake 1981:220-221).

[4] "It is not possible to give a demonstration by shifting from one class (*genos*) of things to another; for instance, one cannot demonstrate something in geometry by means of arithmetic. Hence it impossible to demonstrate by means of

geometry that opposites are studied by a single science, or even that the product of two cubes is a cube; nor can any other science demonstrate something belonging to a different one, except where the subjects are so related that one is subordinated to the other, as optics is subordinated to geometry and harmonics to arithmetic" (Aristotle, *Posterior Analytics*, 75*a*38-9, *b*12-17). The categorical distinction was put in more specifically musical terms by Aristoxenus, a near contemporary of Aristotle, in his *Elementa harmonica*, where he emphasized two species, mutually exclusive, of audible motion. "Aristoxenus presents his account of the [melodic] movement of the singing voice [*phōnē*] by contrasting it with another form of movement within the space or place of pitch. He labels the former as *diastēmatikē*, 'intervallic', the latter as *synechēs*, 'continuous'; and he explains … that movement of the continuous kind is proper to speech. In singing it must be strictly avoided" (Barker 2007:143).

A differentiation of continuous and discrete has, of course, persisted to the present day (as say in analog vs. digital processors), and it has been deeply intertwined with the historical problems of scientific *quantification*. In the scholastic tradition, for example, Thomas Aquinas [1225-1274], pondering the nature of physical space (called 'extension,' from L. *ex+tendere*, to stretch out), related it in general to *quantitas* (quantity, 'muchness'), which was then divided into *quantitas continua* (associated with *magnitudo* [magnitude], *quantitas dimensiva* [dimensional quantity such as volume] and *continuum* itself), and *quantitas discreta (*associated with *multitudo* [multitude, 'manyness'] and *numerus* [number]. However, to the degree that such distinctions were strongly reified — and their modes aligned with different (often 'opposed') disciplines (geometry/arithmetic), different faculties of perception (audition/vision), different subjects (optics/harmonics, acoustics/music), different forms of expression (speech/song), different criteria and techniques of 'quantification' (magnitude/multitude, calculation/measurement) — many modern scientific and mathematical developments were inhibited if not precluded (see especially Neal 2002).

[5] However, "the analysis of the rainbow seems not to have been integral to the science of optics in antiquity" (Smith 1999:149).

[6] Both Ptolemy and Boethius were adept expounders and developers of canonical techniques, the musical 'cutting of the *kanōn*.' Some parallels between canonics and arithmetic were so notable that one could be enlisted to explain the other: "Just as in the division of the musical canon, when a single string is stretched, … with immovable ends, … and the mid-point shifts … in the string by means of the bridge, as in one way after another the aforesaid proportions, arithmetic, geometric, and harmonic are produced, so that the fact becomes apparent they are logically and very properly named, since they are brought about through changing and shifting the middle term in different ways, so too it is both reasonable and possible to insert the mean term that corresponds to each of the proportions between two arithmetic terms [integers], which stay fixed and do not change" (Nicomachus of Gerasa, *Introduction to Arithmetic* II, Chapter xxvii, pp. 845-846).

Allusions to the 'musical proportion' permeated most of Plato's matho-musical discussions (McClain 1978). The notion was stated most explicitly in Plato's *Epinomis*, but then as a consequence of a much deeper history:

> "… all the technical terms of the geometrical theory of proportions have their origins in music … the concepts of 'ratio' (*diastema* or *logos*) and 'proportion' (*analogia*, i.e., 'sameness of ratio'), and even the expressions for operations on ratios, were all developed from considerations about and experiments in the theory of music. … Although [literary sources] report that music and geometry were twin disciplines of the Pythagoreans, only a historical investigation into terminology discloses that the theory of music must have preceded the development of geometry, at least in the creation of its fundamental concepts [the theory of means, for example]. … The oldest stage in the development of Greek mathematics which is still accessible to us is the musical theory of proportion" (Szabó 1978:23, 28).

The long historical hegemony and persistence of arithmological and discrete discourses of musicological and harmonical theories effectively suppressed much of their originally *geometrical* meanings, a significant aspect of the Greek matho-musical heritage, until their revitalization in the 16th and 17th centuries directed toward the measurement of continuous magnitudes and overcoming the prevailing Aristotelian/scholastic ideological distinctions and prohibitions (H.F. Cohen 1984; Pesic 2010, 2014):

> "To Kepler it was obvious that numbers could never provide a criterion for distinguishing consonance from dissonance. … for, in fact, Kepler believed, arithmetic occupies an ontological status quite different from geometry. … in Kepler's view the human mind is shaped to grasp quantities ('*ad quanta intelligenda condita*'). But quantity is not the same as number; rather it is that which is measurable. In Kepler's words: 'Arithmetic is nothing […] but the expressible part of geometry.' Numbers are abstracted from reality; in themselves they do

> not denote anything real. Moreover, numbers are discrete, whereas quantities are continuous. This is why the criterion looked for should be sought among the continuous magnitudes: 'For since the terms of the consonant intervals are continuous quantities, also the causes that distinguish them form the dissonances should be taken from the family of the continuous quantities not from abstract numbers, as a discrete quantity' [Kepler]" (H. F. Cohen 1984:17).

It was especially in the discipline and technology of canonics (theory and practice), with a nearly unbroken history since Euclid's *Sectio canonis* (Creese 2010; Herlinger 2002; Urreitzeita 2010), that a geometrical sensibility about 'musical' phenomena survived:

> [Ancient] "Mathematical harmonics, because it continued to express intervals exclusively as ratios of numbers, never left the realm of arithmetic, but the monochord when it was introduced, brought with it the challenge of constraining the science within the bounds of arithmetic while making use of an instrument whose operation could not be described (not even mathematically) solely within the bounds of arithmetic. The geometrization of harmonics, in its various aspects, was the inevitable result of attempts to make compelling mathematical arguments for the use of numbers and ratios as a language and analytical tool for the investigation of harmonic structures with the assistance of an instrument [the *kanōn*, monochord] which represented numbers and notes as measurable visible distances. … It was the monochord's introduction of geometry that brought with it the problems of representation and exactitude. … It is the high standard set by the argumentative rigour of Euclidean mathematics that seems to have brought the issue to the attention of some harmonic scientists in antiquity, and of course, a solution was impossible: no concrete object can approach the perfection of a mathematical object. … Arithmetic alone could not make the connection between numbers and notes. The sum of developments in what I have called the geometry of sound both furnished that connection more securely than had previously been possible, and also raised doubts about the capacity of instruments to reveal the numerical basis of music. It was in their attempts to minimize these uncertainties that Greek mathematical harmonicists rose to one of the central challenges that faces nearly all sciences that employ instruments" (Creese 2010:356-359).

[7] Simon Stevin [1548-1620], who also embraced the concept of the cyclic octave, "a round in singing," had calculated, algebraically, the string-lengths of an equal-12 division, $(XY)^{n/12}$, of a tensioned string (H. F. Cohen 1984; Rasch 2008).

[8] In addition to the original text, Kepler's procedure is adequately summarized in H. F. Cohen (1984) and Hallyn (1990).

[9] "I say that a mathematical number has no existence proper to itself, and really distinct from the magnitude it denominates, but is only a kind of note or sign of magnitude considered after a certain manner; *viz*. as we conceive it either as altogether incomplex, or as compounded of certain homogeneous parts, every one of which is taken simply, and denominated an unit; or lastly as intimating the ratio it has to other magnitudes, in like manner composed by a certain method. For in order to expound and declare our conception of a magnitude, we design it by the name or character of a certain number, which consequently is nothing else but the note or symbol of such magnitude so taken. This is the general nature, import, and notion of a mathematical number" (Isaac Barrow, *Mathematical Lectures,* 56; *Usefulness of Mathematical Learning*, 41). "For magnitude is the common affection of all physical things, it is interwoven in the Nature of Bodies, blended with all corporeal accidents; I say there is no part of this [Physics] which does not imply Quantity … and consequently which is not in some way dependent on Geometry; Mathematics is adequate … and co-extended with Physics" (Barrow, quoted in Guicciardini 2009:28, note 29).

[10] "Thus, to take the simplest case, a line was thought to be a flowing quantity and was termed accordingly a *fluent*. The rate at which it flowed, the point's velocity, was named the line's *fluxion*. ... The two are clearly related: the velocity of the point will determine the nature of the curve, and a curve of a given nature can only be generated by a point with a certain velocity. Two problems thus arose: how, given a relationship between fluents, can the corresponding relationship between fluxions be determined; and the inverse problem, to determine fluents on the basis of fluxions. The two processes are better known today as differentiation and integration. To tackle the problems Newton first introduced the notion of a *moment*. This was the 'indefinitely small' part by which fluents grew in 'indefinitely small' periods of time" (Gjertsen 1986:214). Newton's 'calculus' made its first appearance in the "October 1666 Tract: To Resolve Problems by Motion" (Newton, *Mathematical Papers* I.2, 7):

" … Proposition 7. Having an Equation expressing y^e relation twixt two or more lines x,y,z, &c: described in y^e same time by two or more moveing bodys, *A*, *B*, *C*, &c: the relation of the velocitys *p*, *q*, *r*, &c may be thus found …" {*differentiation*} …

x A
y B
z C

" … Proposition 8. If two Bodys A & B, by their velocity *p* & *q* describe y^e lines *x* & *y*, & an Equation bee given expressing y^e relation twixt one of y^e lines *x*, & y^e ratio *q*/*p* of their motions *q* & *p*; to find the other line *y*. Could this ever bee done all problems whatever might bee resolved. But by y^e following rules it may bee very often done. (Note y^t $\pm m$ & $\pm n$ are logarithmes or numbers signifying y^e dimensions of *x*.)" {*integration*} …

11 That the 'ancients' had intentionally concealed their 'true' methods of discovery, leaving lost and hidden bits of wisdom to be recovered, was a popular conceit of the day.

12 The "discourse…of strings sounding", Cambridge University Library, *Add. Ms.* 4000, fols. 104^v-113^r and *Add. Ms.* 3958.2. The essay "Of Musick", *CUL, Add.* Ms., 4000, fols. 137^v-142^r (published in Pesic 2006).

13 "… the least number which offers itself in determining all the parts of the monochord … I say that this number is 720, as was shown in [*The Harmony of the World*] Book 3, Chapter 6" (Kepler, *Epitome of Copernican Astronomy* IV.I.4:870 (1620).

14 It is helpful to read these expressions as multiples of aliquot divisions, rather than numerical 'fractions,' for example, 8/9 as 'eight of the $1/9^{th}$ parts of the whole,' or 3/5 as 'three of the $1/5^{th}$ parts of the whole,' and so on.

15 Close inter-relationships between *system*, *symmetry*, and *symphony* were part of the 'classical' heritage of canonics, and they continued to be important ideas in the modern developments of science and mathematics (although not necessarily in actual musical practices), 'symmetry' eventually coming to mean "an invariance of a configuration of elements under a group of automorphic transformations. … the word *symmetry* is used in our everyday language in two meanings. In the one sense symmetric means something like well-proportioned, well-balanced, and symmetry denotes that sort of concordance of several parts by which they integrate into a whole. … In this sense the idea is by no means restricted to spatial objects; the synonym 'harmony' points more toward its acoustical and musical than its geometric applications. *Ebenmass* is a good German equivalent for the Greek symmetry [*συμμετρια*]; for like this it carries also the connotation of 'middle measure,' the mean toward which the virtuous should strive in their actions according to Aristotle's *Nichomachean Ethics*, and which Galen in *De temperamentis* describes as that state of mind which is equally removed from both extremes: *συμετρον οπερ εκατερον των ακρον απεχει.*

The image of the balance provides a natural link to the second sense in which the word symmetry is used in modern times: *bilateral symmetry*, … Now this bilateral symmetry is a strictly geometric and, in contrast to the vague notion of symmetry discussed before, an absolutely precise concept" (Hermann Weyl [1952]1980:3-4).

16 "Almost certainly Newton learnt of the techniques of logarithmic manipulation elsewhere {than directly from the treatises of John Napier [1550-1617] and Henry Briggs [1561-1630], *Arithmitica Logararitmica*, 1624}, perhaps from the tract *De Aequationum Affectarum Resolutione* which [William] Oughtred appended to his *Clavis Mathematicae* [1631] (editions from 1647)" (Derek Whiteside, notes in Newton, *Mathematical Papers* I). Oughtred's *Clavis* was issued in eight editions, with numerous commentaries, during the 17^{th} century; it "became the primary textbook for a whole generation of young mathematicians, some of whom [such as John Wallis] were also taught personally by Oughtred and were to remember him with respect and gratitude all their lives. ... Oughtred had offered the book as 'Ariadne's thread' to lead aspiring mathematicians into the mysteries of classical writings, and in this it succeeded, well beyond the circle of Oughtred's personal pupils. But the real value of the *Clavis* in the end was not as a guide to the past but as an inspiration for the future; Oughtred's key was to open doors to mathematics that Oughtred himself could have hardly imagined" (Stedall 2000:60). Newton was among the supporters of a late (1693) Latin edition of *Clavis*: "Mr Oughtred's *Clavis* being one of ye best as well as one of ye first Essays for reviving ye Art of Geometricall Resolution and composition I agree with ye Oxford Professors [John Wallis and David Gregory] that a correct edition thereof to make it more usefull and bring it into more hands will be both for ye honour of our nation and advantage of Mathematicks" (Newton, *Mathematical Papers* I:16-17). By the time Newton began studying and using logarithms they had been employed for

more than half a century (Bruins 1980), and various applications to 'music' were already known, if not especially abundant (Barbour 1940; Barbieri 1987; Sabaino 2008; Wardhaugh 2008[a,b]; Kuehn and Shepherd 2009).

[17] That is, "… that very good and judicious man, Mr. Oughtred, a man whose judgment (if any man's) may be safely relyed upon" (Newton, *Correspondence* III:364). Oughtred's treatise:

> *The circles of proportion and the horizontal instrument. The former shewing the maner how to work proportions both simple and compound; and the ready and easy resolving of quæstions both in arithmetic, geometrie, & astronomie: and is newly increased with an additament for navigation. All of which rules may also be wrought with the penne by arithmetic, and the canon of triangles. The latter teaching how to work most quæstions, which may be performed by the globe: and to delineat dials upon any kind of plaine.* Invented, and written in latine by W.O. Translated into English, and set out for the public benefit, by William Forster. London: Printed by Augustine Mathewes, and are to bee sold by Nic: Bourne at the Royall Exchange, 1633.

The circular device, multiscalar in concentric circles (some logarithmic), had affixed to its center an *index* to be opened after the manner of 'a paire of Compasses,' hinged so that one 'arme' could be set to the antecedent term of a ratio while the other was set to the consequent term (see Cajori 1994). The index or sector facilitated the transfer of central arc angles from one region of a scale to another and from one concentric circle to another. Operational instructions were given for the rules of ratio and proportion, multiplications, division, continual proportion (or 'progression geometricall'), calculations of roots and powers, the 'measuring of circles, cones, cylinders and sphæres', plane and solid measures, the measuring or gauging of the volumes of vessels, comparisons of sundry metals in quantity and weight, the ordering of soldiers in an rectangular form of battle, 'a collection of the most necessary astronomical operations' for stars in northern and southern latitudes, 'of trigonometria, plaine and sphærical,' and various procedures of dialing.

One case in point of Newton's knowledge of Oughtred's intruments: "*The resolution of y^e^ affected Equation* $x^3 + pxx + qx + r = 0$. Or $x^3 + 10x^2 - 7x = 44$. First having found two or 3 of y^e^ first figures of y^e^ desired roote viz 2∟2 (w^ch^ may been done, either by rationall or Logarithmical tryalls as Mr Oughtred hath taught, or Geometrically by description of lines, or by an instrument consisting of 4 or 5 or more lines of numbers [Gunter's Lines] made to slide by one another w^ch^ may be oblong but better circular) this known pte of the roote I call g , y^e^ other unknown pte I call y, then $g + y = x$. Then I prosecute the Resolution in this manner …" (Newton, *Mathematical Papers* I.3:489-491). The mathematical details of Newton's apparatus, "for approximating the roots of polynomial equations using slide rules" ("oblong but better circular") are nicely presented by Sangwin 2002, who noted that each different equation would require a separate construction and arrangement of the pertinent rules. Edmund Gunter [1581-1626] developed a variety of scientific instruments in the early 17th century (sector, crosse-staffe, quadrant), some of them inscribed with logarithmic scales or 'lines of numbers;' He was especially interested in techniques of stereographic projection and problems of applied mathematics.

[18] http://web.mat.bham.ac.uk/C.J.Sangwin/Sliderules/ocircle.jpg

[19] 'Musical cyclicity' is an ancient and ubiquitous philosophical idea (McClain 1978; Barker 2007:43-57). It was invoked by both Plato (*Timaeus*) and Ptolemy (*Harmonics*; recovered in John Wallis's edition of 1682), and most notably by Simon Stevin, Johannes Kepler and René Descartes in the early 17th century. Circular diagramming of progressions or scales of colors emerged historically much later than those of sounds, although they too were not unknown by the early 17th century (Feller and Stenius 1970; Parkhurst and Feller 1982; Gage 1993). The periodicity of the functions *sin* and *cos* for the number 360 were especially noted in Newton's early papers on trigonometry (*Mathematical Papers* I).

[20] This jargon (with solmization syllables) follows musicological conventions, referring to F-*ut*, C-*ut*, G-*ut*, D-*ut*, A-*ut*, successions of tones which are a 'fifth' (ratio 3:2) distant. In effect, the arrangement shows the one melodic progression, *ut re mi fa sol la fa* [*ut*], transposed to different starting tones. Customary usages referred to such scales as a *gamut* or range (from the Gk. Γ (*gamma*) + *ut*, the lowest tone of a tuning system plus the beginning tone of a melodic (scalar) progression). "By y^e^ helpe of concordant notes y^e^ notes in the Gam ut may be thus tuned" (Newton, CUL, *Add. Ms.* 4000, fol. 105^r^).

[21] This diatonic sequence of intervals, TtSTtST, is slightly different that the 'symmetrical diatonic' pattern, TStTtST, given in TABLE 1 (which is also that of the 'prismatic tuning,' Figures 7, 8, 11), but it is entirely compatible with Newton's general practice. Neither sequence is the same as that in another, but less detailed, tonometric

diagram (CUL, *Add. Ms.* 4000, fol. 109[v], see *back cover*), which shows similar transposition possibilities of the interval sequence **tTSTtST**. Rearrangements of the diatonic order of melodic tones and intervals are *modal* variations, rather than expressions of the *tuning system* as such, and Newton discussed them at some length in his essay "Of Musick". As in the case of Newton's 'polynomial solver' these diagrams required a special construction for each data set or modal pattern under consideration, in contrast to the general and multi-purpose design of Oughtred's "circles of proportion."

[22] Historically, the equal-53 division has been credited to the mathematician Nicolaus Mercator [1620-1687], a major synthesizer of logarithmic thinking (*Logarithmotechnica*, 1668) in the 17[th] century (Helmholtz [1887]1954; Barbour 1940, 1951; Wardhaugh 2008[a,b]). Mercator wrote on the solutions of triangles using the logarithmic functions of sine, cosine, tangent and cotangent at one degree intervals (*Trigonometria sphaericorum logarithmica* (1651). He also wrote on 'music' and had corresponded with young Newton and other mathematicians of the day. Mercator was an advocate of Keplerian astronomy, including the 'harmonic/systemic' considerations of consonant and dissonant motions and the correspondences of Kepler's musical-geometrical partitioning of an encircled tensioned string. Mercator's *Institutionum Astronomicum,* and *Hypothesis astronomica nova* (London, 1664) were the among the sources for Newton's learning about Kepler's laws of celestial motion and the thesis that the orbits of the planets were elliptical.

Mercator's works on 'music,' which circulated in scribal publications, were preoccupied with enumerative questions, such as "*how many* times the tonus major (8/9) is conteyned in the diapason (1/2)." An answer was obtainable by following a 'rule' of *calculation*: 'find the logarithms of each of the ratios (as fractions) and divide the greater by the smaller.' Thus he determined, by $\log(2/1)/\log(9/8) = 0.301029995/0.051152522$, that approximately 5.884949192… 'major tones' were contained in an 'octave' (see Wardhaugh 2008[a,b]). In contrast, Newton's interests and methods were oriented to a philosophically different universe of questions and consequences, not by counting multitudes of something, but with determining *how much* some observed or defined magnitude was in excess (or deficiency) another according to some systematic 'rule' of *measurement*, an emphasis that included adherence to the techniques of metrical geometry in 'Euclidian' canonics.

By about the third century BCE, Euclid had demonstrated (*Sectio canonis*, prop. 9; see Barbera 1991:147-149), using canonical principles, that a continuous magnitude (*diastema*, distance, interval) comprised of six successive 'major tones' $(9{:}8)^6$, a geometric progression (ratio $8^6 : 9^6 \propto 262{,}144 : 451{,}441$), exceeds (is greater than) a 'duple' interval (ratio $1{:}2 \propto 262{,}144 : 524{,}288$) by exactly one 'comma, the exact magnitude of their 'difference' (ratio 531,441 : 524,288) [Euclid's own numbers]. That also meant that $9^6 >$ twice 8^6 by 7153. Such determinations were oriented to measuring the relationships of actual magnitudes (intervals, distances) rather than feigning an analysis of nature through an 'equality of numbers.' "Equations are expressions of arithmetical computation and properly have no place in geometry…", Newton said, and, giving references to Euclid's *Elements* V where geometrical progression ('continued proportion') is defined, he also wrote: "Of the Signification of some Words and Notes. Article I. By Number we understand not so much a Multitude of Unities, as the abstracted Ratio of a Quantity, to another Quantity of the same Kind, which we take for Unity" (Newton, *Arithmetica universalis*, *Mathematical Papers* V, pp. 54-491) — as in the ratios of partial string-lengths to a whole reference quantum "1 or 720". Newton was, of course, master and commander of techniques of calculation (including logarithms), but he cautioned against conflating the philosophical differences of algebras and geometries. His rules were scales of measurement applied in the geometrization of natural phenomena, including the metrical geometry of a tuning system (**TABLE 1**) and the shape of the prismatic spectrum (**Figure 7**).

Two aspects of a 'Euclidian' technique, not readily apparent in Mercator's practice of determining 'how many' but central to Newton's TONOMETRY and COLORIMETRY, are 'orderliness' and 'muchness.' Orderliness derives from the *progressive* construction of scales or rules, and musical tones and intervals are often expressed in the ordinal terms of their position in a succession ('seconds,' 'fifths,' etc.), and the components of such constructions also provide for the quantification of 'how much' one magnitude is greater or lesser another. In the manuscripts that make up Newton's "discourse … of strings sounding" he wrote in a distinctly 'Euclidian' manner of measurement, summarizing, for example, that "By this table it may appeare that … [a given "musicall" interval or tone] is higher/lower by y[e] [n^{th}] parte of a note than it would bee were y[e] musicall chord divided in a geometricall progression" (Newton, ULC, *Add. Ms.* 4000, fol. 106[v]). In one instance he stated:

"*A [musical] 3[d] minor is higher of y[e] just* [geometrical 3[d] minor] *by y[e] 5/64[th+] pte of a* [geometrical] *note*".

Some paraphrase and mathematical ‘translation’ of Newton’s terse and idiosyncratic shorthand will be helpful:

> ‘A *musical* interval of a ‘3^{d} minor’ [ratio **6:5**] is higher [larger or more distant from the “Ground sound” (720)] than a *just* [even or *geometrical* interval] of a ‘3^{d} minor’ [ratio $1:(\sqrt[12]{2})^3$] by the 5/64^{th+} part of a *just* ‘whole note’ [that is, the ratio $1:(\sqrt[12]{2})^2$].’ More informally, this says that on a ‘musically’ ordered scale the ‘third’ tone minor) above the ground tone is higher than the ‘third’ tone minor on a ‘geometrically’ ordered scale, by a specifiable ‘muchness,’ a ‘part of a note.’

Significantly, Newton’s term “just” in this passage meant nothing at all musicological but rather mathematically ‘even’ or ‘geometrically equidistant,’ much like the present meaning of ‘justified margins’ of text on a page. We can apply (as Newton did) a measuring ruler graduated into 12 “geometrically progressionall” parts to an octave (ratio **1:2**) for the whole string “1 or 720”, so that each of its “Equidistant ½ notes” has the value $1:2^{1/12}$. The ‘logarithmic’ measure of the interval of a “musicall 3rd minor” (ratio 720:600 ∝ 6:5) is $(12/\log 2)(\log 6 - \log 5]) = 3.15641287$; alternately $6:5 \propto 1:(\sqrt[12]{2})^{3.15641287}$. In contrast, the corresponding “just” interval, that of a “geometrical 3rd minor”, is $1:(\sqrt[12]{2})^{3.000000}$, and the indexes (exponents) of the two intervals show that the “musicall” interval is greater (“higher”) than the “geometricall” one: 3.15641287 > 3.0000000. Newton went on to measure *how much* those two intervals differ, that is, the one is greater than the other by the addition of *y^{e} 5/64^{th+} pte of a* [geometrical] *note*”. A geometrical or equidistant *whole* “note” is two geometrical “½ notes”; it has the rational value $1:2^{2/12}$ so the specified increment of excess is 5/64^{th+} of 2 (or 10/64^{th+}, decimally 0.15625^{+}). Newton’s notations required that that increment be added to the index (exponent) of the “geometrical 3rd minor”, yielding 3.15625^{+}. Returning to the canonical language of relative string lengths, the whole reference quantum **720** can be ‘divided’ to obtain the partial lengths indicating the “musicall” and “just” intervals and tones being compared.

$$720/(\sqrt[12]{2})^{3.15641287} \cong 720/(\sqrt[12]{2})^{3+10/64+} \text{ or } 720/(\sqrt[12]{2})^{3.15625+} > 720/(\sqrt[12]{2})^{3.000000}$$

$$600.000000 \cong 600.005645 > 605.445419$$

‘600’ (which marks the “musicall” tone, a ‘third’ in order) is *more distant* from **720**, the reference quantum or “Ground sound”, than 605.+… (which marks the “geometricall” tone), “as number is to an unit”, so that the interval 720:600 > 720:605.445419 by the addition of 5/64^{th+} part of a whole “just note”.

23 An anonymous pamphlet, entitled “The Musicall Compass” and dated 1684, had also referred to the ‘circularity’ of the octave. It included a paper device, “apparently intended to help place the frets on a musical instrument,” which was divided into 31-equal parts, similar to the proposals of Christiaan Huygens — *Le cycle harmonique* [1691] and *Novus cyclus harmonicus* [1724]. … “The ‘musical compass’ … was imitated in unpublished work by Brooke Taylor [1685-1731] in the 1710s, … [by] similar devices with multiple moving parts, apparently intending to facilitate the comparison between different equal divisions of the octave” ([2008[a]:170, 181). Newton did not list an equal-31 division, nor some others proposed in the 17th century (such as Joseph Sauveur’s equal 43, Juan Caramuel Lobkowitz’s equal-69, or William Brouncker’s equal-17). Unlike these writers, Newton’s mathematical interests emphasized general methods or techniques of analysis and description over the advocacy of any particular musical systems or arrangements.

24 “The Original or primary colours are Red, Yellow, Green, Blue, and a Violet-purple, together with Orange, Indico, and an Indefinite variety of Intermediate gradations” (Newton, “New Theory about Light and Colours,” 1672). Newton’s occasional designation of prismatic colors as ‘primary’ and ‘primitive,’ and metaphorical references to painting and pictures, contributed, historically, to numerous confusions of his theory of light and spectral colors with those of the mixing of pigments and dyes. However, a comingling of terminologies from different disciplines is hardly unusual, and is often characteristic in the early stages of scientific inquiry and theory development (on the topic see Shapiro 1994; Gage 2008). However, for Newton ‘primary’ spectral colors were not ‘few’ but innumerably (infinitely) ‘many’: “…by ‘primary’ he [Newton] means the physically irreducible, elementary components of light, which are separated by their physical property of refrangibility. This concept of ‘primary’ differs from … those in the arts tradition of pigment mixing, where the primary colors are those elements out of which all other colors can be made and which cannot themselves be made from any others” (Shapiro 1994:618). Newton’s phrase reads that ‘original or primary colors are some named ones *together with* an indefinite (un-named and un-numbered) variety of other ones.’ He also referred to the ‘named’ or ‘listed’ colors as the “principall” or “more eminent species” of colors, ‘colors’ proper ‘originating’ in human perception and cognizance.

25 Newton claimed to have achieved a length to breadth ratio of the visible spectrum of as much as 70 to 1, maximizing the separation of the homogeneal (simple) rays of light and minimizing their comingling: "…the small Circles may at pleasure be diminished, whilst their Centers remain in their places. … I made the Breadth of the Image *pt* to be forty times, and sometimes sixty or seventy times less than its Length. … For the composition [compounding] of heterogeneal Rays is in this Light so little, that it is scarce to be discovered and perceived by Sense…" (Newton, *Opticks* 1704]1979:69-70).

"The same colours in *Specie* with these Primary ones may also be produced by composition. For a mixture of *Yellow* and *Blew* make *Green*, of *Red* and *Yellow* makes *Orange*; or *Orange* and *Yellowish-green* makes *Yellow*. And in general, if any two Colours be mixed, which in the series of those, generated by the Prisme, are not too far distant one from another, they by their mutual alloy, compound that Colour, which in the said series appeareth in the mid-way between them. But those that are situated at too great a distance, do not so. *Orange* and *Indico* produce not the intermediate *Green*, nor *Scarlet* and *Green* the intermediate *Yellow*" (Newton, "New Theory about Light and Colours," 1672).

26 Another possible description of the dimensions of the colors is *arithmetic*. Assume the spectrum's extended length ***XY*** to be composed of the subtractive differences of the 'musical' string-length numbers between **720** and **360**; the unequal 'spaces' occupied by the principal colors, as they appear to direct inspection of the spectrum, then measure: **360 = 80 (Violet) + 40 (Indigo) + 60 (Blue) + 60 (Green) + 48 (Yellow) + 27 (Orange) + 45 (Red)**. This indicates the *difform* or non-uniform distribution of the spectral colors, wherein, for example, the extent (interval) of **Indigo** (40/360) is different and greater than the extent of **Orange** (27/360). But an arithmetic specification is less informative than Newton's *rational* approach, wherein **Orange** (432:405 ∝ 16:15) and **Indigo** (640:600 ∝ **16:15**) are the 'same,' a description that includes all the parts of an orderly succession comprising the whole shape and system of spectral colors. Newton also preferred the rational and musical-*geometrical* characterization of the spectrum insofar as it "embodied linear law of prismatic dispersion" (Shapiro 1979). But, tyupically, he continued to account for and to describe the *shape* of the spectral image in multiple ways: "Finally, from these [musical-geometrical measurements] the proportions of the sines of refraction are determined by a mechanical procedure. Specifically, for glass contiguous to air, when the sines of the outermost rays in each side are as 68 to 69, divide the intermediate unit in the ratio of the parts of this image; and there will result 68, $68^1/_8$, $68^1/_5$, $68^1/_3$, $68^1/_2$, $68^2/_3$, $68^7/_9$, 69 for the sines pertaining to the boundaries and ends of the seven individual colors with respect to a common sine of incidence 44 … . Moreover, for the sines pertaining to the rays where the colors are the most perfect of their own species, the intermediate numbers between these numbers can be used" (Newton, *Optical Papers, Optica* II, *Lecture* 11, c. 1671-2, pp. 547-149). Consequently, the variety of descriptions, all used by Newton, of the elongated shape of the prismatic spectrum—canonical, geometrical, rational, arithmetic, trigonometric—were all coordinate.

27 This casual statement, which was not published, seems to confuse the mixing of sounds and lights with the mixing of substances (see Shapiro 1993, 1994). The intervals mentioned, 'note or tone' (ratio 9:8) and 'fift' (ratio 3:2), were prominent in the musicological conventions of the day, but here they are nominal entities at best, and not measured or quantified distances as elsewhere in Newton's research. The relationships between intervallic qualities, affects, and magnitudes (for either tones or colors) were asserted, rather dogmatically, without question or explanation: "… the fift being next above the Key, to w[ch] it adds so much sweetnesse y[t] should this fift bee omitted in any song, y[e] Key would emparte its name & nature to some sound w[ch] hath a fift above it. And since all harmony without a fift is flat, therefore the Key must necessarily have a fift above it" (Newton, "Of Musick", CUL, *Add. Ms.* 4000, fol. 138[r]). Various 'musicalized' schemes for mixing colored substances were prevalent in 17[th] century discourses (see especially Gage 1993), but the confused and inconclusive observations that Newton listed here were reason enough to avoid publication.

28 In responding to Hooke, "Newton introduced a new idea which marked a great improvement upon Hooke's treatment of light. Led by the analogy between light and sound, he [Newton] realized that in an account of the properties of light in wave terms, the vibrations (or waves) corresponding to the various colours must differ from one another in some definite property, their bigness or wave-length. The heterogeneity of white light would then consist in the infinite plurality of the wave-forms corresponding to the component colours. But it must be realized that, for Newton, the component wave-forms (the 'unequal vibrations') must exist with their characteristic regularities *before* they are separated by refraction. In other words, the original compositeness of white light (whatever be the terms, wave or otherwise, in which it is conceived) must have a definite physical meaning. The blending of the various vibrations to compose white light did not mean, in Newton's suggestion, that the waves would lose their individual characteristics in such a mixture; these waves, as they are sent out by the various parts of the luminous body (which according to their

different sizes, figures and motions, excite different vibrations in the ether) might cross one another, but they may not combine, or alter one another; they must exist as differentiated elements of a heterogeneous mixture" (Sabra 1981:278-279). (For more on Hooke's ideas see Gal 2002; Gouk 1980, 1999; Kassler 1995; Kassler and Oldroyd 1995; Mamiani 2000; Sabra 1981; Wardhaugh 2008[b]).

Newton carried the concept of the 'original heterogeneity' of both light and sound on through to their modes of propagation. In a short work, *De aere et aethere* (c. 1679), he described the propagation of sound by variable pulsations (undulatory motions) or oscillations of the rarefaction and compression of air. "In writing about the nature of things I begin with heavenly bodies and among them with the one most available to the senses, to wit, the air, in order that I may follow where the senses lead". 'Air' was understood to consist of physical 'particles' of diverse sizes, the more subtle of which formed a continuum with the 'ether.' "And just as bodies of this Earth by breaking into small particles are converted into air, so these [grosser] particles can be broken into lesser ones by some violent action and converted into yet more air which, if it is subtle enough to penetrate the pores of glass, crystal, and other terrestrial bodies, we may call the spirit of air, or the aether". Most abstractly and generally, in *Principia mathematica*, Book II, Propositions 47–50, Newton stated that 'the velocity of propagation of a pulse in an elastic fluid is directly proportional to the square root of the elasticity and inversely proportional to the square root of the density.'

[29] A slightly different phrasing for parts of this passage occurred in Newton's manuscripts and in the version published by Thomas Birch in *The History of the Royal Society*, Vol. 3, 1757, pp. 262-263 (reprint: A. Rupert Hall and Marie Boas Hall, (eds.), Johnson Reprints, 1968; and also in *PLNP*, pp. 192-193).

Discrepancies between these versions are mostly minor, but a few are mentionable and in two cases crucial:

(1) "I suppose that as bodeys of various sizes, densities, or <u>tensions</u>…" [*Correspondence*]
"I suppose, that as bodies of various sizes, densities or <u>sensations</u>…" [*PLNP*]

(2) "…yet the *differences* of the Observations were but little, especially toward the red end, & takeing <u>meanes between those differences that were the length of the Image</u> (reckoned … by the distance of the Centers of those Semicircles [xy], …to Sound the tones <u>in an eight</u>" [*Correspondence*]

"…yet the *differences* of the observations were but little, especially towards the red end, and taking <u>means between those differences, that were, the length of the image</u> (reckoned … by the distance of the centres of those semicircles [xy], …to sound the tones <u>in the eighth</u>" [*PLNP*]

The *Correspondence* version makes the canonical theme more explicit, referring to string 'tensions' rather than 'sensations,' and defining string 'length' according to the different terms denoting its extremes, relative to which all other locations are medial or intermediates along a continuous length as "meanes between", rather than implying 'averages' of divergent observations.

Another version, in manuscript, helps clarify why Newton found it useful to compare the range of the whole spectral dispersion (*PT*, *xy*), including all the intermediate variety of colors from the least to most refracted, to the rational magnitude of an 'octave' (ratio 2:1); and notably there it was the 'soul' (Gk. *psuche*) rather than the 'sensorium' that was said to be affected.

> "Newton … maintained that when light corpuscles strike the retina, they excite vibrations in the aether contained within it. These vibrations are propagated into the brain where they "… affect the soule wth a sensation of various colours according to their various proportions, something after the manner that various sounds are produced by various proportions of the vibrations of the Air. The harmony and discord also w^{ch} the more skilfull Painters observe in colours may perhaps be effected & explicated by various proportions of the aethereall vibrations as those of sounds are by the aëreall. To which end I would suppose the vibrations causing the deepest scarlet to be to those causing the deepest violet as two to one; for so there would be all that variety in colours w^{ch} wthin the compasse of an eight is found in sounds, & the reason why the extreames of colour Purple & scarlet resemble one another would be the same that causes Octaves (the extreames of sounds) to have in some measure the nature of unisons" ([Newton, *Add. Ms.* 3970, fol. 528^{v}] Shapiro, note 28 in Newton, *Optical Papers*, 1984:546).

Another term for 'octave,' Gk. *diapason*, meant 'through all,' that is, it referred to *all* the variety of tones and intervals, all the intermediate degrees of sound, occurring between the extremes tones of the relative interval ratio $2:1$ ($\propto$ Newton's reference length $720:360$). Any 'halvings' or 'doublings' of that interval were deemed to be periodic replications of that variety, introducing no new or different musical or sonorous materials. Mathematically however, any 'unit' (as some selected unitary or whole thing) could be taken to be a reference quantum. And Newton pursued that possibility elsewhere (in *Opticks* [1704]1979:210-227), adjusting the ratio according to some empirical determinations of the rational magnitude of the interval (or 'distance') of the *most* to *least* refrangible rays of light, as they occurred in the periodic phenomenon of 'Newton's rings' in thin transparent bodies. There the *most*:*least* ratios were given as 'greater than **3:2** and less than **13:8**, most frequently as **14:9**, and better estimated as **14¼:9** [∝ **57:36**] or **14⅓:9** [∝ **43:27**].' Additionally, Newton 'seemed to collect' a distance "very nearly as the six lengths of a Chord which sound the notes in a sixth Major [ratio 5:3], *sol*, *la*, *mi*, *fa*, *sol*, *la*. But it agrees something better with the Observation to say that … the limits of the seven Colours, red, orange, yellow, green, blue, indigo, violet in order, are to one another as the Cube Roots of the Squares of the eight lengths of a Chord [a Musical Chord, a Monochord] which sound the Notes in an Eight, *sol*, *la*, *fa*, *sol*, *la*, *mi*, *fa*, *sol*, that is, as the Cube Roots of the Squares of the Numbers, 1, 8/9, 5/6, 3/4, 2/3, 3/5, 9/16, 1/2."

Using the very same canonical and tonometrical technique that Newton applied in his "discourse … of strings sounding" (c. 1665), the relative sizes of these *most*:*least* ratios can be compared, measuring them by a rule calibrated for twelve "equidistant" intervals per 'octave' (ratio **2:1**), that is, by calculating 'a number of the ratio' $(12/\log 2)(\log most - \log least)$ for each of them and listing the results in order of size:

—	3:2	7.019550…	
	14:9	7.64959…	
	57:36	7.955580…	
	$(2:1)^{2/3}$	8.000000	
	43:27	8.056527…	
	[8:5	8.136862…	a "sixth Minor"]
—	13:8	8.405276…	
	5:3	8.843587…	a "sixth Major"

Newton's reference to a "sixth Major" (ratio **5:3**) could have been a mistake, since it is too large an interval, and it lies outside the originally listed range of observations. But perhaps it was mentioned to re-'musicalize' the discussion according to Newton's usual 'diatonic' practice. Metrically, a "sixth Minor" (ratio **8:5** interpolated above) might have been a better choice than a "sixth Major". But while a "sixth Minor" was specified in Newton's tuning system (**TABLE 1**), it cannot be directly expressed in the 'diatonic' order of intervals in the prismatic tuning, **TStTtST**; nor could it be given in terms of the solmization syllables which, in all of Newton's uses, are only 'diatonic.' The cumulative intervals for the six tones *sol*, *la*, *fa*, *sol*, *la*, *mi* (**T + S + t + T + t**) comprise a "sixth Major". Ultimately, Newton reconciled the various rational measurements with the full 'diatonic' musical structure of the spectral dispersion, by the specification 'Cube Roots of the Squares of the Numbers.' Any string-length rationally shorter or longer than Newton's original "1 or 720" (for the reference 'octave' **720:360**) can be partitioned either musically or geometrically; any string-length is simply a whole or unitary magnitude, partitionable whatever way one desires. Placing the prismatic tuning on a different string-length effects no more than a relative transposition of its given set of tones and intervals to a higher or lower pitch range. The overall extent of the interval Newton defines above (by cube roots of squares) is the *most*:*least* ratio $(1 : ½)^{2/3} \propto (2^{2/3} : 1) \propto (1.587401052 : 1)$, an reference interval smaller that an 'octave' (ratio **2:1**). In his early manuscripts (c. 1665) Newton listed (by string-length ratios) the "musicall" intervals **720:432** ∝ **5:3** (a 6^{t} major) and **720:450** ∝ **8:5** (a 6^{t} minor). He also tabulated a "geometricall" interval ***720:453.571578***, according to "y^{e} Logarithmes of those strings, or distances of y^{e} notes" and "A string (720) divided into 12 (geometrically progresionall) parts y^{t} it may sound y^{e} 12 exact [equidistant] ½ notes in an eight" (Newton, *Add. Ms,* 4000, fol. 105^{v}). And he listed that "geometricall" interval's 'exponential' value as '8,' meaning that its size measures exactly 8 of the 12 equidistant parts of an 'octave,' that is, it was a partial length of the whole "1 or 720", an 'equal-12 sixth minor' $1:2^{8/12} \propto 1:2^{2/3}$. To confirm its relative magnitude in the list above calculate $(12/\log 2)(\log 720 - \log 453.571578) \cong 8$. The intermediate partitions given here for this interval are the same as those indicated in **TABLES 1, 2, 3** and **Figure 7**, above. Newton's expression 'cube roots of the squares' is reminiscent "of Kepler's Third Law of planetary motion [the 'harmonic law'] which notes the proportion between the cube of the distance of a planet from the sun and the square of its period" (Pesic 2006:297; see also Pesic 2014; Kepler [1619]1997: V.3, p. 411; Shapiro 1993; Gingerich 1993; Gal 2002).

[30] Most canonical practices were designed to isolate this variable and consider it independently of other factors. In discussing the building and various methods of dividing monochords, Vincenzo Galileo ([1581]2003:118) explained "Why the Ancient Musicians Arranged the Monochord on a Single String: The ancient Greek musicians systematized [canonical divisions] on a single string in order that they could hear and understand exactly the quality and quantity of

the [tonal] intervals and to avoid the deception that might result from having two or more strings owing to the [physical] variability of strings and various circumstances that may always intervene" (see also Creese 2010; Dostrovsky 1975).

[31] "In the scientific tradition ... a clear distinction is made between words [or numbers or diagrams] and things and between literal and metaphorical language. The occult tradition does not recognize this distinction: Words are treated as if they are equivalent to things and can be substituted for them. Manipulate the one and you manipulate the other. Analogies, instead of being, as they are in the scientific tradition, explanatory devices subordinate to argument and proof, or heuristic tools to make models that can be tested, corrected, and abandoned if necessary, are instead, modes of conceiving relationships in the universe that reify, rigidify, and ultimately come to dominate thought. One no longer uses analogies. One is used by them. They become the only way in which one can think or experience the world" (Vickers 1984:95).

For example, "A universe without Newtonian dynamics is now literally unthinkable, as is a rainbow without seven colours" (Gouk 1999:255); see also Topper 1990; Wardhaugh 2008[a,b]).

[32] "I think that tastes, odors, colors, and so on are no more than mere names so far as the object in which we locate them are concerned, and that they reside in consciousness. Hence if the living creature were removed, all these qualities would be wiped away and annihilated" (Galileo Galilei, *The Assayer* [1623]).

[33] In a recent and informative work, *Newton and the Origin of Civilization*, Buchwald and Feingold (2013) singled out one part of Newton's "Hypothesis" "to explain colours" — the reference (marked *…* above) to multiple observers and observations at different times, places and circumstances in the analysis of the prismatic spectrum — for being especially telling about his general attitude towards the senses, one they characterize as 'skeptical and unreliable as a basis of scientific knowledge.' Their thesis is very extensive, a *tour de force*, but it runs contrary to much of what I have explored here "in Experiments, where Sense is Judge". They have proposed that Newton's 'methods' were 'solutions' to the peripatetic and scholastic doctrine that the unaided senses do not penetrate to knowledge of the essences of physical realities, and that sensory experiences, therefore, are improper and untrustworthy pathways to scientific and epistemological truths. Newton did not disagree with the first part of that notion: "…we certainly do not know what is the substance of any thing. We see only the shapes and colors of bodies, we hear only their sounds, we touch only their external surfaces, we smell only their odors, and we taste their flavors. But there is no direct sense and there are no indirect reflected actions by which we know innermost substances, …" (*Principia mathematica* (3rd ed.), 'General scholium,' 1999:942). (And compare Locke: "That the *Species of Things to us, are nothing but the ranking them under distinct Names, according to the complex* Ideas *in us*; and not according to precise, distinct, real *Essences* in them, is plain from hence; … Our faculties carry us no farther towards the knowledge and distinction of Substances, than a Collection of those sensible *Ideas*, which we observe in them; which however made with the greatest diligence and exactness, we are capable of, yet is more remote from the true internal Constitutions, from which those Qualities flow, than … a Countryman's *Idea* is from the inward contrivance of that famous Clock at *Stratsburg*, whereof he only sees the outward Figure and Motions." [Locke, *Essay* III, vi, 8-10]). However, neither Locke nor Newton concluded or inferred, as did Descartes for example, that the human senses were inherently and categorically inadequate, defective and error prone, with little relevance for science ("it is the mind which sees, not the [biological] eye," said Descartes, so that only what the 'mind's eye' 'reflects on' can be the ground of true knowledge (Descartes [1937]2001:108; see also Wilson, Atherton in Levin 1997; Hill 2007). Quite differently, Newton accepted a basic and ordinary sensorial worthiness, apprehending the infinite diversity and plenitude of perceptual phenomena, naturally apparent; and it was for him especially important to understand the roles of the sensory phenomena of sounds and colors in natural philosophy. But here, more specifically, is what Buchwald and Feingold had to say:

> "The eye cannot be relied upon to judge the physical character of even of the purest white, or indeed of any color whatsoever. Only a device that has the power to destroy the phenomenon can reveal what it truly is, and the senses have no such abilities. The eye, for example, operates less well than a telescope for seeing far, or than a microscope for seeing near, and so it may be considered to be a defective instrument when responding to its appropriate stimulus. But Newtonian senses are not merely defective, as Descartes, Huygens, or Hooke thought, for they cannot probe the aspects of things that lie beyond their ken. The eye, for example cannot unpack the physical nature of color.
>
> Newton accordingly came to think that the senses, and especially vision, inevitably and irremediably deceive. The senses can be artificially stimulated to produce perceptions that do not differ from those engendered by

external objects. Or they can be overstimulated (e.g., vision by the sun). This was known to Newton's mechanically minded contemporaries, or indeed to scholastics. Aristotelians considered these sorts of effects to be illusions engendered by an improperly working sense. Mechanical philosophers, on the other hand, thought like Newton; whatever stimulated a sense in a given way would necessarily produce the same effect, with the inevitable consequence that even a properly working sense could deceive unless compensated by an appropriate device. Newton's early distrust of sensory information was accordingly exacerbated by experiments which convinced him that what seems a pure, unitary white to vision must be an occult congeries of colored lights. Only a device that did not improve, but that actually replaced, the ordinary operations of vision could detect the true physical nature of light.

> The senses are accordingly thrice problematic for Newton: they may be weak or defective even in their normal operations; they are unreliable in that they may—in uncontrollable ways—exhibit effects that are not a result of the stimulation of external objects under investigation; and finally, they are inherently unable to probe the hidden structure of phenomena. Weak, unreliable, and inadequate senses, and make the design, implementation, and interpretation of experiments, especially quantitative ones, a difficult and problematic affair. Pursued in the manner of Huygens or Hooke, namely by attempting to reach as good a number as possible through skillful work and (in the case of Hooke) elaborate apparatus, experiments could never wipe away the taint of sensory caprice.

> It is probably not a coincidence that Newton's very first quantitative endeavor in the laboratory engendered a method to use against itself …" (Buchwald and Feingold 2013: 89).

To support their case, Buchwald and Feingold quoted, without reference to the canonical context, only part of the "Hypothesis" passage given above "to explain colors" (between *…*), interspersed with an interpretative commentary:

> "Newton not only took averages among numbers that should, on theoretical grounds, be the same, he took averages of averages. … he stated explicitly that in one case at least he had made several discrepant measurements, and that he had formed the 'mean' among them. This exception is particularly significant because it reveals Newton's skeptical attitude toward the ability of the senses to provide reliable data. … "Why had Newton resorted to an 'assistant'? … Poorly discriminating eyes and the prejudicial effect of prior knowledge together demanded an unbiased observer, equipped with a better color-perceiving sense. But no one's visual apparatus is for Newton truly reliable; it must always be errant, and his way to compensate error was via multiple replication. … What did he do then with all of these, which were difficult and inaccurate, because 'the just confines of the colours are hard to be assigned, because they pass into one another by insensible gradation'? … Even so, he continued, 'the *differences* of the observations were but little, especially towards the red end, and taking means between those differences, that were, the length of the image … was divided in about the same proportion that a strings is, between the end and the middle, to sound the tones in the eighth' " (Buchwald and Feingold 2013:93; on this version see NOTE 29, above).

The authors continued with outlining Newton's 'solution' to the problems of sensory 'inadequacy':

> "Newton's 'mean'—the average—was the weapon with which he slew the inevitable dragons of sensual error. It was a most paradoxical weapon for the times, because it amounted to a method by which error seems to be reduced by committing it repeatedly. No such method appears elsewhere at the time, and it would certainly have seemed odd, to say the least, to most practitioners of the period. Newton himself was reluctant to use it in print when he presented actual numbers—except *in this one place* [*…* in the passage "to explain colours" in "An Hypothesis" of 1675], where he exposed the method he had utilized when taking a group of measurements [*emphasis* added]. Why advertise the method? The reason is not hard to find: Newton knew perfectly well just how difficult it was to pin down the regions of the spectrum filled by one of his seven colors; it was inevitable that anyone else attempting to do so would encounter the same problem and might well object to a bald claim that the lengths are such-and-such solely on the grounds that he 'found' them to be so—as he asserted in respect to the arithmetic progression for his colored rings. His 'assistant' with exceptionally good and unprejudiced eyes, though he indubitably existed, also amounted to a marvelous rhetorical stand-in for the reader himself, a presence in the laboratory with Newton that went beyond the detailed narrative accounts, such as those produced by Boyle, and which were designed to bring the reader virtually into the author's experimental space. For here, Newton had an active participant in situ—more than a participant, in fact, but the very author of the observations on which Newton rested his claims. He had, as it were, created a physical displacement followed by

a virtual transposition, having replaced himself with the assistant, and then the assistant with the reader" (Buchwald and Feingold 2013:93-94).

There are, of course, many kinds of inquiries where sensory variability, errors or deceptions can become epistemologically problematic, and Buchwald and Feingold are exploring one of those, that of Newton's enlistment of scientific astronomy in the historical critique and reconstruction of the very 'origins of human civilization.' However, while the senses are limited and potentially fallible so too are scientific instruments, logics and languages; and it would be a mistake to assume that battling 'dragons of sensory error' comprised any manner of general doctrine for Newton. Ironically, in fact, Buchwald and Feingold's citation from '*this one place'* was just on those very topics (sounds and colors) where Newton originally and repeatedly asserted sensory reliability. Newton's rhetorical ploy (postmodern sophistries on the 'role of the reader' aside) simply re-asserted his reliance on consensual 'perceptual acquaintances' (sensory presences to inquisitive observers) that provided sufficient and appropriate *data* for thinking philosophically about the natural phenomena of colors. Assuredly, empirical strategies involving sensations and perceptions did not comprise the whole of the natural philosophical endeavor, but whatever their limitations, being themselves 'natural' and belonging to 'physics' in its broadest compass, vision and audition could hardly be dismissed or denigrated. Some styles of 'Newtonianism' have indeed insisted on an 'anaesthetized' or senseless scientific project (see NOTE 2, above), but Newton's own inquiries, especially in the domains of sounds and lights (tones and colors, audition and vision), were in pursuit of establishing acceptable agreements (co-relations) between sensibles and reasonables [such as Mersenne's laws], rather than 'replacing' the one with the other. That notion was prominent in the history of canonics, for example, "the thesis that reason does not contradict sense perception is a recurring motive throughout Boethius's treatise" … "The goal of harmonics is defined … by Ptolemy, such that there is nothing irreconcilable between the ears and reason. According to Ptolemy, the harmonic scholar directs his activity in such a way that what the sense estimates, reason weighs; accordingly, reason searches out ratios to which the sense expresses no objection" (Boethius, *Fundamentals of Music*, p. xxxiv; p. 165).

Everyone, of course, reads and writes from some certain perspective. My interest here is that the bulk of Newton's "Hypothesis" passage "to explain colours" is technically 'canonical,' and it is in those explicit terms that Newton would have had his readers 'understand him best,' especially in regard to the meaning of 'means': "You will understand me best by viewing the annexed figure (the "Pillar of colours" diagram, a schematic monochord, *kanōn*) and its attendant musical-(geo)metrical specifications, "…takeing *meanes between those differences that were the length of the Image* (*reckoned* not by the distance of the verges of the Semicircular ends [*PT*], but *by the distances of the Centers of those Semicircles* [xy], or length of the Streight Sides as it ought to be) [the image] was divided in about the same proportion that a String [xz] is, between the end [x] & the middle [y], to Sound the tones in the eighth [octave]" [*emphasis* added].

In the first place, Newton's 'method' (too large a word, so better: his original canonical technique preceding the "Hypothesis" statement) of viewing, analyzing and describing the shape of the prismatic spectrum ("some years past", *Optical Lectures*, *Optica* II, "Lecture 11," 1671-2), expressed no distrust of sensory perceptions nor did it presume intrinsic errors in sensory judgments. Ordinary vision, simply seeing or looking at the prismatic *spectrum* displayed on a wall ("at first a very pleasing divertissement, to view the vivid and intense colors") precipitated a challenge to the prevailing theory of refraction by noticing (detecting) the spectrum's unexpected oblong shape. For Newton the relevant geometry was 'in the nature of things,' not, as say for Descartes, solely 'in the mind,' where intellectual geometry was posed (and imposed) as a 'corrective lens' and antidote to the bodily afflictions of 'bad' eyesight.

In contrast to Buchwald and Feingold's reading of Newton's 'means' in 'this one place' as some sort of statistical or numerical average of a 'group of measurements' or observations (and he did use such on various occasions), according to Newton's canonical specification 'means' are medial or intermediate magnitudes, indicating the boundaries of the "more eminent species" of colors which lie between the defining 'extreme' magnitudes delimiting the spectrum's whole length (actual PT or relative xy). Hence, he wrote of "takeing meanes between those differences that were the length of the Image", that is, a *length* 'reckoned as the relative distance *xy*' (that is, the 'difference' of the lengths *xz* and *yz*). Newton initially arrived at and determined those medial locations using and explicitly trusting his senses. In the original 'musical-geometrical' description of the spectrum (*Optical Lectures*, "Lecture 11"), he wrote that he proceeded "with as much care as I could, trusting not only my own senses, but also relying on the judgment of others …". Moreover, "because it agrees with the phenomena [of spectral colors] very well", Newton found it relevant to repeat the reasonable and geometrically measured description of the spectrum, in whole and parts, that ensued from that trust and reliance throughout the next half-century. Similarly, in addressing the criticisms of Hooke and Huygens, Newton frequently prioritized sensory information, choosing "to speak of *Colours* according to the information of our Senses" and emphasizing "it is much better to believe our senses, informing us, than to suppose…".

Most technically, Newton's "meanes" in this context are relative 'lengths' (string or line), continuous magnitudes, sometimes (but not necessarily) signified by numbers; they are partial lengths *intermediate* to the *longest* (*xz* or **720**) and *shortest* (*yz* or **360**) lengths under consideration (see **Figure 7**). To clarify, consider the first datum of Newton canonical technique in "Lecture 11" *Optical Lectures*, pp. 537-549:

> "To investigate [the shape and dimensions of the spectral image's relative length *xy*] with greater accuracy I marked with a pen the places where the most perfect colors of their kind and their boundaries fell on a transverse piece of paper, and having frequently repeated such observations and compared them to one another, I finally drew these [5 listed} conclusions one by one:
>
> > [The first 'conclusion':] "Blue and violet on the one side, and green, yellow and red on the other side bisected the image [the length *xy*], such that the boundary of green and blue (which I may call sea green [*thalassinum*]) occupied its middle. …
>
> "…When I observed these [locations, places] with as much care as I could, trusting not only my own senses, but also relying on the judgment of others (because of the extreme difficulty in precisely distinguishing the colors' boundaries and places of greatest perfection), I drew the dimensions of the image according to these determinations…"

Repeated and continued visual examination of the 'shape' of the spectrum led to a refined diagram and description which fully integrated diverse perceptual criteria — the locations of the boundaries separators of colors, of each color's 'center,' and the relative extent of each color in an orderly succession — into a single model. All the medial positions, of color boundaries *and* the central locations of their 'greatest perfection,' within the whole spectral length were similarly determined by direct visual inspection, according to Newton's report. All the sensory data were brought into a non-problematic agreement with a rational account. The first bisector ("the boundary of green and blue") was the point H (*sol*) in **Figure 7** and the "Pillar of colours" figure (above). A geometrical bisection of a length, decided by closely consistent judgments of the locations and color differences of 'blue' and 'green' within the spectrum's overall length, defined xy like Kepler's 'two signal termini,' was unrelated to any technique of statistical (numerical) averaging, even while that initial data point between the extremes x and y indicated a 'mean' (medial, intermediate, middle) color ('sea green') in the overall progression of colors.

With Newton's supposing XZ to be the length of a musical string double the length of the visible spectrum (XY), the location of H bisecting XY became re-characterized such that the following ratios obtained (as part is to whole, as number is to unit, for the length "1 or 720"): the length HZ is medial to the *longer:shorter* extreme lengths (XZ and YZ) in the specific canonical ratio of $XZ{:}HZ \propto 720{:}540 \propto$ 4:3 (with its octave complement of $HZ{:}YZ \propto 540{:}360 \propto 3{:}2$), In other words the 'geometrical' position of the bisector H (**540**) becomes the 'arithmetic mean' of the spectral length when $XY \propto 720{:}360$. Another of Newton's initial 'observations' noted the perceived extent of the 'green', the interval HI, to be "about 1/6th part of the entire …length"; consequently $XZ{:}IZ \propto 720{:}480 \propto$ 3:2 and $IZ{:}YZ \propto 480{:}360 \propto 4{:}3$, the position I (**480**) becoming the' harmonic mean' when $XY \propto 720{:}360$.

Eventually, Newton assembled all the observations (positional determinations) on the spectral image into a synoptical and canonical statement which captured and described the overall systemic rationality of the appearance of the prismatic spectrum, fulfilling the stated goal of "Lecture 11" (1671-2) to describe "the shape of the colored image formed by light flowing through a narrow, round hole into a dark room and then passing through a prism, and to examine diligently the dimensions of each of its colors, their distances from one another". The refined description and diagram were expressed in musical-geometrical terms, such that "everything appeared just as the parts of the image occupied by the colors were proportional to a string divided" [as **TStTtST**, **Figures 5**, **7**; "Pillar of colors"]. The more "eminent species" (and names) of colors were rationally delineated, in order, and "each of these, when the entire mass of colors fell on the whole figure, was respectively contained within these individual parts. Approximately in the middle of these parts each color appeared the most brilliant and intense of its own species". And ultimately, "everything turns out proportionate to the quantity of green with a more refined symmetry".

Newton's refined, musicalized and geometrized description and diagram of the prismatic spectrum provided an optimal integration of a variety of perceptual criteria: the boundaries of the more eminent colors (differentiating hue), their middle positions of 'greatest perfection' (minimizing spectral mixing or compounding), exhaustiveness of the whole and orderliness of its parts, and so on. In general, there was a good agreement, sufficient for Newton purposes, between the perceptual data and the rational description. The 'musical' model of the visual appearance of the prismatic spectrum, diligently informed by perceptual judgments and rationally constructed, said Newton simply, "agrees with the phenomena very well".

Newton certainly acknowledged that, in consequence of the continuous (non-discrete) gradation of spectral colors, precise locations of the boundaries and centers of the principal colors were somewhat indeterminable, and he provided an alternative metrical description. All the mean lengths being considered are 'means proportional' and relative to defined extremes; they are expressed as ratios of lengths, but thus far only the *difform* ratios of a musical tuning have been mentioned. Newton also noted that selecting the second, third, fifth, seventh, ninth and tenth of eleven (*uniform*) mean proportional lengths (a continued geometrical proportion) between the shortest and the longest lengths yielded the 'geometrical' divisions of a monochord (as shown in **TABLE 2** and **Figure 7**). And, he wrote, "this [geometricall] distribution will also seem to fit the colors' expanses sufficiently well. For such minute differences that occur between this [geometricall] and the above [musicall] distribution can produce errors hardly visible to the keenest judge." The discrepancies in the descriptive metrics between the "musicall" and "geometricall" distributions vary between 1.4% and 0.2% of the total extent of the visible spectrum. But the 'significance' did not lie in the 'numbers' or some manner of calculation, but rather in the judgments that decided whether perceived differences constituted relevant differences for understanding the problem in hand. In this case, small range perceptual variations, differences that arose from trusting one's senses but were non-distinctive (negligible), were set aside in order to attend to the differences that did matter. This is a question of judgment, both perceptual and intellectual, Newton expressed no desire here to 'slay a dragon of sensory error by numerical averaging' (in fact there were no numbers to 'average'), partly because the discrepancies are more in the descriptions than in the "hardly visible" perceptions; and Newton appealed to a principle of minimal noticeable differences, as supplied by 'perceptual acquaintance,' to structure the information pertinent to the questions at stake.

Moreover, perceptual variations in the 'information of the senses', whether great or small, for different times, places, circumstances, individuals and cultures, contemporary and historical, were accepted by Newton as ordinary facts of human experience. Nothing pertinent to natural philosophy warranted treating such variations as inherently 'unnatural' or 'deceitful' or in 'error.' As, said Newton, "No man wonders at the indefinite variety … of all … naturall things [the usuall objects of our senses]… it being almost impossible for any sort of things to be found without some contingent variety". Nature's providential plenitude was manifest and immediately present for consideration. In his student notebooks he wrote: "The senses of diverse men are diversely affected by y^e^ same objects according to y^e^ diversity of theire constitution. To them of Java pepper is cold. … To one pallate y^t^ is sweete w^ch^ is bitter to another. The same thing smells gratefully to one displeasanly to another. Objects of sight move not some but cast others into an exstasie. Musicall aires are not heard by all w^th^ alike pleasure. The like of touching" (Newton, in McGuire and Tamny 2002:382, 396). Newton was later little interested in such (predominantly emotional) 'affections.' They were abandoned in favor of more abstract perceivables pertaining to natural shapes, distances and motions. Even so, he continued with the ordinary awareness and expectation that "the senses of diverse men are diversely affected by the same objects".

Following Newton's injunction that "'tis better to put the Event on further Circumstances of the Experiment", his direct reliance on the "information of our Senses" "in Experiments, where Sense in Judge" was also exercised (without any 'musical' [geo-]metrics) throughout the *Opticks* (1704). Here, for example, from BOOK ONE, PART II, PROP. II, pp. 122-125): "*All homogeneal Light has its proper Colour answering to its Degree of Refrangibility, and that Colour cannot be changed by Reflexions and Refractions*", not only were small observational variations disregarded, but perceptual judgments supplied all the distinctive *data* (information) supporting the proposition. First, by means of prisms and lenses, Newton maximally…

"…separated ['severed'] the heterogeneous Rays of Light from one another [see NOTE 25, **Figure 6**, so that] the Spectrum *pt* [or *xy*] did in its Progress from its End *p*, on which the most refrangible Rays fell, unto its other End *t*, on which the least refrangible Rays fell, appear tinged with this Series of Colours, violet, indigo, blue, green, yellow, orange, red, together with all their intermediate Degrees in a continual Succession perpetually varying. So that there appeared as many Degrees of Colours, as there were sorts of Rays differing in Refrangibility" (*Opticks*, p. 122). "For the Composition [comingling] of heterogeneal Rays is in this Light so little, that it is scarce to be discovered and perceiv'd by Sense…" (Newton, *Opticks*, p. 70). "And because refraction onely *severs* them, & *changes* not the bignesse or strength of the ray, thence, it is, after they are once well severed, refraction cannot make any further changes in their colour" [as perceived] ("Hypothesis").

Then, Newton selected for viewing "one very little part" of that whole range of spectral light by allowing only its transmission, through a small circular aperture, to be cast onto a white paper, that minute part (say of the 'red' light) being about 1/60th the whole extent of the spectral dispersion. Subsequently, he …

"…found that the Spectrum [the little image] formed on the Paper by this ['little part' of transmitted] Light was not oblong as when 'tis made [as in the large image of the whole spectrum] by refracting the Sun's compound Light, but was (so far as I could judge by my Eye) perfectly circular, … Which shows, that this light is refracted regularly without any Dilation of the Rays" (*Opticks*, p. 73). "If any [little] part of the red Light was [further] refracted, it remained totally of the same [to sense] red Color as before ... no other new color was produced by that [or any continued] Refraction. Neither did the Colour any way change by repeated Refractions, but continued always the same red entirely as at first. The like Constancy and Immutablity [of and to sense] I found also in the blue, green, and other Colours" (*Opticks*, pp. 122-3).

Newton illuminated small objects (such as flies and printed letters) alternately with "homogeneal" (simple) and "heterogeneal" (compound) lights, and he then viewed those …

"…same Objects, through the same Prism at the same distance, and in the same situation, [so that] there was no difference, but in the Light by which the Objects were illuminated. … In the homogeneal Light I placed Flies, … and viewing them through a Prism, I saw their Parts as distinctly defined, as if I had viewed them with the naked Eye. The same Objects placed in the Sun's unrefracted heterogeneal Light, which was white, I viewed also through a Prism, and saw them most confusedly defined, so that I could not distinguish their smaller Parts from one another. I placed also the Letters of a small print, one while in the homogeneal Light, and then in the heterogeneal, and viewing them through a Prism, they appeared in the latter Case so confused and indistinct, that I could not read them; but in the former they appeared so distinct, that I could read readily, and thought I saw them as distinct, as when I view'd them with my naked Eye." (Newton, *Opticks* [1704]1979:74).

…if I looked through a Prism at any Body [object] illuminated with any [very little] part of this homogeneal light, … I could not perceive any new Colour generated this way. All Bodies illuminated with compound [heterogeneal] Light appear through Prisms confused, … and tinged with various new Colours, but those illuminated with homogeneal Light appeared through Prisms either less distinct, nor otherwise colour'd, than when viewed with the naked Eyes. Their Colours were not in the least changed by the Refraction of the interposed Prism. I speak here of a sensible Change of Colour: For the Light which I here call homogeneal, being not absolutely homogeneal [that is, it is a small but still variable part], there ought to arise some little Change of Colour from its [small] Heterogeneity. But, if that Heterogeneity was so little as it might be made by the said Experiments [those demonstrating the maximum separation of rays], that Change was not sensible, and therefore in Experiments, where Sense is Judge, ought to accounted none at all". … From all of which it is manifest, that if the Sun's Light consisted of but one sort of Rays, there would be but one Colour in the whole World, nor would it be possible to produce any new Colour by Reflexions and Refractions, and by consequence that the variety of Colours depends upon the Composition of Light" (Newton, *Opticks* [1704]1979:123-124).

Throughout his researches into the nature of light and colors, Newton relied on the sensory and perceptual detection of changes (or absence thereof) in shapes and colors, under varying and controlled circumstances, for determining and furnishing the basic *data* informing a philosophical understanding. Such evidence (visual appearances) was, "in Experiments, where Sense is Judge", comprised of maximally noticeable differences: oblong/circular shape, different/not-different hue, change/no change of color, clarity/confusion of focus, and so on. Small range or minimal variations, or near undetectable 'insensible gradations' in that data, were treated as differences that did not make for a significant difference in formulating the overall conclusions. In these writings there is no suspicion of the inadequacy of the senses to make the relevant distinctions, nor is there any particular foreboding of an observer-independent or perceiver-independent 'physics' in which instruments 'replace' normal vision ("Only a device that did not improve, but that actually replaced, the ordinary operations of vision could detect the true physical nature of light," according to Buchwald and Feingold 2013). However, *looking at* or *viewing* objects through prisms and lenses, however 'extraordinary,' continues to be 'seeing' according to all the normal functioning of vision. In the works of the 17th century natural philosophers theories and appreciations of *perception* (audition, vision, etc.) were integral to the 'physical' theories of sound, light, etc. — "All the management of our lives depends on the senses" (Descartes, *Optics*).

Lastly, Newton accorded the human senses and sensory information quite central and important roles in his late thoughts about nature (Gk. *phusis*) and the natural processes of generation (becoming, being). And, much as in Locke's

Essay Concerning Humane Understanding (1690), he sought clarification of central semiotic/epistemological problems: "...the Extent and Certainty of our Knowledge ... has so near a Connexion with Words, that unless their force and manner of Signification were first well observed, there could be very little said clearly and pertinently concerning Knowledge" (Locke, *Essay* III, ix, 21). Newton took up a similar theme in a *scholium*, following the initial *Definitions* in the *Philosophiæ Naturalis Principia Mathematica* (1687) and preceding the *Axioms of the Laws of Motion*:

"Thus far it has seemed best to explain the senses [understandings] in which less familiar words {such as 'centripetal force,' 'quantity of motion'] are to be taken in this treatise. Although time, place, space, and motion are very familiar to everyone, it must be noted that these quantities are popularly conceived solely with reference to the objects of sense perception. And this is the source of certain preconceptions. To eliminate them it is useful to distinguish these quantities into absolute and relative, true and apparent, mathematical and common." …

… [Since "absolute places", that is, "parts of absolute space"] "… cannot be seen and cannot be distinguished from one another by our senses, we use sensible measures in their stead. For we define all places [absolute or relative] on the basis of the positions and distances of things from some body that we regard as immovable, and then we reckon all motions with respect to these places, insofar as we conceive of bodies and being changed in position with respect to them. Thus, instead of absolute places and [absolute] motions we use relative ones, which is not inappropriate in ordinary human affairs, although in philosophy abstraction from the senses is required. For it is possible that there is not a body truly [absolutely] at rest to which [absolute] places and [absolute] motions may be referred."

The principle 'preconception to be eliminated' was that of conceiving 'quantities' "*solely* with reference to the objects of sense perception", allowing then 'quantities' to be *also* conceivable with reference to intellectual (conceptual, mathematical, intelligible) objects. Insofar as 'abstraction from the senses is required,' there needs be senses to abstract from (as well as to confirm or infirm the abstractions). In Newton's "Pillar of Colours" discussion in *Opticks* [1704]1979:125-132, with the spectral "Image of the Sun" visibly displayed on a wall or paper, he instructed readers to "conceive" its systematic organization in whole and parts as a musical string rationally and proportionally divided (see above, **Figure 7**) while asserting that the perception and the conceptualization were in sufficiently good agreement. In the easiest of terms Newton advised '*look* at the spectrum and *think* of it this way.' Moreover, 'conceiving sensibly' and 'conceiving conceptually' are both grounded in human sentience and action as are 'thinking popularly' and 'thinking philosophically.' In this *scholium* Newton was urging close attention to the framing of the referents of the words and definitions being used.

In the 'General scholium' added to the second and third editions (1713, 1726) of *Philosophiæ Naturalis Principia Mathematica* (1687), Newton continued:

"Every sentient soul, at different times and in different organs of senses and motions, is the same indivisible person. There are parts that are successive in duration and coexist in space, but neither of these exist in the person of man or in his thinking principles, and much less in the thinking substance of God. Every man, insofar as he is a thing that has senses, is one and the same man throughout his lifetime in each and every organ of his senses. God is one and the same God always and everywhere … It follows that all of him is like himself: he is all eye, all ear, all brain, all arm, all force of sensing, of understanding, and of acting, but in a way not at all human, in a way not at all corporeal, in a way utterly unknown to us. As a blind man has no idea of colors, so we have no idea of the ways in which the most wise God senses and understands all things. … No variation in things arises from blind metaphysical necessity, which must be the same always and everywhere. All the diversity of created things, each in its place and time, could only have arisen from the ideas and will of a necessarily existent being. But God is said allegorically to see, hear, speak, laugh, love, hate, desire, give, receive, rejoice, be angry, fight, build, form, construct. For all discourse about God is derived through a certain similitude from things human, which while not perfect is nevertheless a similitude of some kind" (Newton, *Principia mathematica,* 1999:941-943).

Similar themes were expressed in the open questions of Newton's *Opticks* (editions after the Latin of 1706 and second English of 1717).

"…the Soul of man is [not] the Soul of the Species of Things carried through the organs of Sense into the place of its Sensation, where it perceives them by means of its immediate presence, without the Intervention of any third thing. The Organs of Sense are not for enabling the Soul to perceive the Species of Things in its Sensorium, but only for conveying them thither; and God has no need of such Organs, he being everywhere present to the Things themselves" (Newton, *Opticks* [1704]1979:Query 31, p. 403).

"…the main Business of natural Philosophy is to argue from Phænomena without feigning Hypotheses, and to deduce Causes from Effects, till we come to the very first Cause, which certainly is not mechanical; and not only to unfold the Mechanism of the World, but chiefly to resolve … such …Questions {as} …whence is it that arises all that Order and Beauty which we see in the World? … How came the Bodies of Animals to be contrived with so much Art, and for what ends were their several Parts? Was the Eye contrived without Skill in Opticks, and the Ear without Knowledge of Sounds? … Is not the Sensory of Animals that place to which the sensitive Substance is present, and into which the sensible Species of Things are carried through the Nerves and Brain, that there they may be perceived by their immediate presence to that Substance. And these things being rightly dispatch'd, does it not appear from Phænomena that there is a Being incorporeal, living, intelligent, omnipresent, who in infinite Space, as it were in his Sensory, sees the things themselves intimately, and thoroughly perceives them, and comprehends them wholly by their immediate presence to himself: Of which things the Images only carried though the Organs of Sense into our little Sensoriums, are there seen and beheld by that which in us perceives and thinks" (Newton, *Opticks* [1704]1979, Query 28, pp. 369-370).

Whatever significance these late comments might have within the airy precincts of philosophical and theological thought, there can be little doubt that Newton considered the senses and sensory information to be of paramount importance in natural philosophy. He emphasized "every man insofar as he has senses", rather than 'insofar as he reasons, eats, walks upright, or possesses a divine 'soul'. The privileged site of the articulation of corporeal human being and incorporeal human being is sensorial (even while the specific character of that 'place', is largely unknown). The place, *topos*, called the 'sensorium' is a place of sensory 'presence' or 'perceptual acquaintance,' where the senses are 'intelligencers' (messengers, informers), preludes of thought and knowledge and action, without which natural philosophy would be impossible. It is also a place in which sensory motives originate as well as one that designates a principal philosophical 'topic:' 'the motion[s] of light and sounds being among the general topics that are most fundamental for natural philosophy' (*Principia mathematica* III). For Newton, this 'place' was essentially organic and 'vital' (and not exclusively spiritual or material or mechanical or chemical or electrical or psychological). In some writings Newton associated it with the Hebrew notion of *maqom*, 'the place in which we live and move and have our being' (Brooke 1989:173; see also Yolton 1983; de Pater 2005; Wolfe and Gal 2010). Newton's natural philosophy strove to be holisitic, to fully embrace the 'psychical' along with the 'physical,' to pursue human understanding according to, *inter alia*, the formulations of rational mechanics while not neglecting thereby its 'main business:' "… to resolve … such … Questions {as} …whence is it that arises all that Order and Beauty which we see in the World?"

To conclude, much of Newton's explorations of the metrical geometries of audial and visual 'spaces,' were grounded on an explicit trust and reliance on sensory determinations and judgments in supplying basic empirical data "in experiments, where Sense is Judge" — abstractly formable as scientific 'laws' pertaining to the relationships of light and vision, sound and audition (seeing and hearing, color and tone) — related, in some manner or other, to the arts of music and painting — and intimately associated with significant philosophical puzzles pertaining to the "sensorium" (whatever that might be). These tenor and trend of such notions are incompatible, even deeply contrary, to any general doctrine in which "the senses, and especially vision, inevitably and irremediably deceive," which holds a "distrust of sensory information," where scientific reasoning is ever afflicted and beset by "the taint of sensory caprice," where methods are constructed to slay "the inevitable dragons of sensual error" and mechanical instruments are designed to "replace" vital and normal sensory functioning.

[34] That visual perception does not actually work directly by optical and luminous means, but in ways more akin to audial/acoustical principles, was another 'Keplarian' observation. In *The Optical Part of Astronomy* [1604], Kepler emphasized the empirical fact no light enters posterior the *tunica retina* in the eye of a perceiver. Interior the mind/brain there is no light, and 'seeing' (as intellection) works quite well 'in the dark,' by principles not unlike those of 'hearing.'

> "For, by the laws of optics, what can be said about this hidden motion, which, since it take place through opaque and hence dark parts and is brought about by spirits which differ in every respect from the humours of the eye and other transparent things, immediately puts itself outside the field of optical laws? And yet it is this motion that brings about vision, from which the name Optics is derived; and so it is wrong to exclude it from the science of Optics, simply because, in the present limited state of our science, it cannot be accommodated in Optics."

The brain is not a mirror, it does not 'reflect' or 'mirror' external realities, and ideas are not pictures: "… despite a heavy use of and even some influence by the optical treatises, the majority of seventeenth- and eighteenth-century writers did not consider the mind to be a mirror of nature" (Yolton 1984:222). Rather, the brain, indeed the whole body, was thought to be an 'instrument' (Gr. *organon*): "But to the purpose: this variable composition of man's body hath made it as an Instrument easy to distemper; and therefore the Poets did well to conjoin Music and Medicine in Apollo; because the Office of Medicine is but to tune this curious harp of man's body and to reduce it to Harmony" (Francis Bacon, *The Twoo Bookes of the Proficience and Advancement of Learning: Divine and Humane*, 1605). All the sensory perceptions, the foundations of empirical knowledge, were thought to come about by the detection and interpretation of *motions* (vibrations, pulsations, locomotion, translation, etc.). This doctrine was endemic to the 17th century, and it much favored 'musical/acoustical' models of perception and cognition (see Crombie 1990, 1996; Dostrovsky 1975; Frank 1990; Kassler 1982, 1984, 1995, 2001; Yolton 1983, 1984; Rousseau 1989, 1990, 2004; Wallace 2003; Wardhaugh 2008[c]; Wolfe and Gal 2010; Erlmann 2010; Buchwald and Feingold 2013). In "An Hypothesis Explaining the Properties of Light" (1675) Newton had also commented that the sensations of light were engendered by "vibrations (like those of Sound in a trunk or trumpet,) [that] run along … through the optic Nerves into the sensorium (wch Light itself cannot doe)". In explaining 'vision,' for another example, Descartes wrote: "it is necessary to think that the nature of our mind is such that the force of the movements in the areas of the brain where the small fibers of the optic nerves originate cause it to perceive light; and the character of these movements cause it to have the perception of color: just as the movements of the nerves which respond to the ears cause it to hear sounds, and those of the nerves of the tongue cause it to taste flavors, and, generally, those of the nerves of the entire body cause it to feel some tickling, …" (Descartes [1637]2001:101). For all Newton's progress in optical research, undemonstrated 'causal' conjectures ('hypotheses') such as these remained for him open questions rather than conclusions (e.g., *Opticks*, 1704: *Queries*). Nor did he hold that all the senses were the same; despite some metaphorical similarities each was understood to be contingent upon different natural phenomena. Philosophical orientations pertaining to the nature of acoustics/audition and optics/vision are complex and diverse, both historically and throughout the 17th century (see especially Levin 1997), and Newton was usually prudent in avoiding premature commitments as to their similarities and relationships.

[35] The actual centers of gravity of each of the given segments of the circular COLORIMETER lie on the bisectors of the respective arcs and they are located at less than **R=1** from the center ***O***. For arcs **DE**, **AG**, **CD** (all ratio **8:9** or **T**) the centers (***p***, ***s***, and ***x***) are **.9168709114 R**; for **FG**, **AB** (both ratio **9:10** or **t**) the centers (***r*** and ***t***) are **.9331035684 R**; and for the intervals **EF**, **BC** (both ratio **15:16** or **S**) the centers (***q*** and ***v***) are **.9745439462 R**.

[36] 'Classical' cosmologies propounded a variety of associations between system, symmetry, and symphony, and thereby implicated relationships (diversely 'spiritual' and 'material') between 'gravity' (weight) and 'harmony.' Newton suspected that ancient philosophers had provided a historical charter for his new scientific formulations, and to illustrate that he wrote up a set of classical *scholia*, selectively gleaned and paraphrased from available texts. The *scholia* were proposed to accompany specific propositions in a second edition of the *Principia mathematica*, Book III, *The System of the World*: "Up to this point I have explained the properties of gravity. I have not made the slightest consideration about its cause. However, I would relate what the ancients thought about this" (Newton, in Schüller 2001:239). Newton's *scholia* manuscripts were recovered from the estate of David Gregory, who had earlier noted (see above) Newton's idea that "the amount of the colours in pure (?white) light is as the harmonic divisions of a string." In brief:

Proposition 8: "If two globes gravitate toward each other, and their matter is homogeneous on all sides in regions that are equally distant from the center, then the weight of either globe toward the other will be inversely as the squares of the distances between the centers."

> *Scholium*: "The ratio by which gravity decreases as the distance from the planet increases was not sufficiently explained by the ancients. They appear to have concealed this ratio using the harmony of the celestial spheres, whereby they portrayed the sun and the remaining … planets … as Apollo with the seven-stringed lyre and measured the intervals between the spheres through the tone intervals. … Similarly, by the oracle of Apollo … [the] Sun is named *King of the seven-tone harmony*. Through this symbol they indicated that the sun acts on the planets with its force in the same harmonic ratio to the different distances as that of the tensile force to strings of different length, i.e., in a duplicate inverse ratio to the distances. … Along the way the philosophers loved to arrange their mystical language such that they offered the masses quite inappropriate posts for the sake of derision and hid the truth behind such language" (Newton, in Schüller 2001:235, 237; see also Tonietti 2000).

COMMENT: Superficially, Newton had suggested that the old symbolisms implicated basic acoustical proportionalities of stretched strings: fundamental frequency proportional to length, proportional the square root of tension, and inversely proportional to the square root of mass per unit length. More significantly, Newton's philosophical interests were entangled in broad, difficult and contemporary cosmological and theological questions, among them accounting for the non-circular motions of planets according to the multiple and variable forces compounding those motions. His late 'force' law, the inverse square law (ISL) of mutual gravitational attraction, was one piece of the puzzle, but a unique ISL..

> "…was never 'discovered'. It had been suggested, speculated and hypothesized by different people, for different reasons, in different contexts, to fulfill different goals. … There were as many inventions of ISL as there were reasons leading to it, significations attached to it and applications of it. …
>
> … Early suggestions that the sun's influence on the planets diminishes by the square of the distance are to be found in medieval optics, and were reported by arguments of the same structure as [Robert] Hooke's: light is distributed in concentric spheres around its source (the sun in the case of the heavens). Since there is a set 'quantity' of light, the larger the sphere, the smaller the 'density' of light [that] will be falling on each point in it. And since the surface of a sphere is proportional to the square of its radius, the amount of light received by any given light is diminished proportionally to the square of its distance from its light source. Or, in other words the amount of light falling on the planets is inversely proportional to the square of their distance from the sun" (Gal 2002: 169, 176-177; see especially Gal and Chen Morris 2005; 2006).

In an essay "On Circular Motion" (c. 1665-7) Newton derived one version of an ISL from Kepler's 'third' law, that is, the one first given in *The Harmony of the World* V.3 (1619): "… it is absolutely certain and exact that the ratio which exists between the periodic times of any two planets is precisely the ratio of their 3/2th 'powers' of their mean distances." And Newton derived in *De Motu* ["On the Motion of Revolving Bodies," c. 1685]): "Since in the primary planets the cubes of their distances from the Sun are reciprocally as the squares of the number of revolutions in a given time: the endeavors of receding from the Sun will be reciprocally as the squares of the distances from the Sun." Kepler had formulated an ISL for planetary motions in direct analogy with the illuminative power of light. He proposed that a solar motive 'power' (*virtus motrix*) moved all the planets about *and* that the light of the sun itself conveyed the very proportions (harmonies) by which those motions occurred: "As regards *movement*: the sun is the first cause of the movements of the planets and the first mover of the universe, even by reason of its own body. … As regards the *harmony of the movements*: the sun occupies that place in which alone the movements of the planets give the appearance of magnitudes harmonically proportioned" (Kepler, *Epitome of Copernican Astronomy* IV.I: 855). These themes, co-implicating 'harmony' and 'gravity' and 'illumination' in terms of an ISL, were continued in Ismaël Boulliau's [1605-1694] *De natura lucis* (1638) and *Astronomia Philolaica* (1645), an author well known to and cited by Newton. In his early paper on fluents and fluxions (see Note 6 above), Newton went so far as to use the phrase 'Rays of Gravity.' Philolaus of Croton, a teacher of Plato's teacher Archytas, was widely acknowledged, amongst all the heliocentric theorists from Copernicus onwards, as the progenitor of a 'harmonic/geometrical *light* centric cosmology'. Boulliau wrote:

> "As for the power by which the Sun seizes or holds the planets, and which, being corporeal, functions in the manner of hands, it is emitted in straight lines throughout the whole extent of the world, and like the species of the Sun, it turns with the body of the Sun; now, seeing that it is corporeal, it becomes weaker and attenuated at a greater distance or interval, and the ratio of its decrease in strength is the same as in the case of light, namely, the duplicate proportion, but inversely, of the distances that is, $1/d^2$."

Proposition 9: "In going inward from the surfaces of the planets, gravity decreases exactly in the ratio of the distance from the center [as long as the matter of the planets be of uniform density]."

> *Scholium*: "Thales believed every body to be animate and concluded this from the magnetic and electrical attractions. … And … he had to attribute the attraction of gravity to the soul of the matter in question. … Plato said that the soul was an essence moving itself; Xenocrates, a number moving itself; <Artistoteles> called it entelechy>; Pythagoras and Philolaus, harmony; Democritus, a spirit implanted in the atoms having such freedom of movement that it permeated the body. Therefore Plato, guided by Pythagoras' teachings … <knew> that there can be no star constellation brought together without these numbers [namely the musical numbers], in his Timaeus he introduced the world-soul by interweaving these numbers … Thus the world-soul, which stirs to motion the body of the universe that we now witness, must be interwoven with those numbers which produce musical harmony. … In impelling the bodies of the world, [the soul] produces tones which are separated from each other by unequal intervals" {after Macrobius, primarily} (Newton, in Schüller 2001:241, 243, 245).

COMMENT: Here Newton alluded to the concept of the 'world-soul' (Gk. ψυχὴ, *psuche*; L. *anima mundi*), an infamous *matho-melodic* construction in Plato's *Timaeus*. It was said to be suffused throughout the spheroidal cosmos from center to periphery (that is, *canonically* 'stretched, partitioned, and pattern woven') so that everything contained therein would be 'bonded' together organically, made orderly, enlivened and set in motion, and partake of both rationality and sensibility (for technical details see especially McClain 1978; Gersh 1996; Barker 2007; Creese 2010). The 'world-soul' was Plato's autonomous principle and origin (*archē*) of all motions in the universe, physical and psychical. She was motion's efficient cause (αιτια κινεσεως), herself motion and perpetually in motion–"moved throughout her whole self" (*Timaeus* 37a). The idea was, consequently, of considerable interest to the scientists of the 17th century, whether for the study of the uniformly accelerated motions of Galileo's second 'new science,' the motions of pendulums, or the compounded motions of the celestial bodies. In the 17th century there were those who thought this 'soul' or 'spirit' to be the 'prime mover,' the very essence of 'deity.' In Newton's *General Scholium* to the 2nd ed. of *Principia*, he pointedly distanced his own 'metaphysics' from "those for whom God is the world-soul", while he favored, alternately, the existence (paraphrasing here) of 'a certain very subtle spirit pervading gross bodies and lying within them; [which], by its forces and actions, particles attract one another and cohere contiguously at short distances, electrical [electro-magnetic] bodies act at greater distances repelling and attracting, light is emitted, reflected, refracted, inflected, and heats bodies; and all sensation is excited … by the vibrations of this spirit.' Of course, this does not exclude the 'vibrations of this spirit' being, as it were, also interwoven with those phenomena that generate 'harmonies' ("For it is well known, that Bodies act upon one another by the Attractions of Gravity, Magnetism, and Electricity; and these Instances show the Tenor and Course of Nature, and make it not improbable that there may be more attractive Powers than these. For Nature is very *consonant and conformable* to her self" (Newton, *Opticks* [1704]1979:376; emphasis added). "Certainly idle fancies ought not to be fabricated recklessly against the evidence of experiments, nor should we depart from the analogy of nature, since nature is always simple and ever consonant with itself [*sibi consona*]" (Newton, *Principia mathematica* [1687]1999:III:795).

37 Newton generally avoided making premature commitments as to the specific nature of these 'forces,' whether they were of waves or particles, electrical or magnetical, vibrational or chemical or spiritual. He did conjecture, cautiously. "I knew that the *Properties* of *Light* were in some measure capable of being explicated by many different Mechanical *Hypotheses*. And therefore I chose to decline them all, and to speak of Light in *general* terms, considering it abstractly, as something or other propagated every way in straight lines from luminous bodies, without determining, what that Thing is; … And for the same reason I chose to speak of *Colours* according to the information of our Senses, as if they were Qualities of Light *without* us" (Newton, "Mr. Newton's Answer to Some [Robert Hooke's] Considerations," 1672). The phrase "information of our Senses" is telling of a major trend of 17th century natural science; deemphasizing mechanistic and causal explanations, the senses, essential to empirical investigations, were increasingly understood to be instrumental in organizing and conveying 'information' about the world. They were the agents of 'perceptual acquaintance' and the media of 'sensible commerce with nature' (Deely 1982; Yolton 1984; Crombie 1990; Wallace 2003; Salter 2010). Their function, differently than their physical or neurophysiological nature *per se*, was thought to be much like that of the Queen's secret agents ('intelligencers') abroad: "It is true that all the Senses are Intelligencers to the Soul less or more; for tho they have their distinct limits, and proper Objects assigned them by nature; yet she is able to use their service even in the most abstracted of Notions, and Arbitrary institution. [Moreover,] Nature seems to have fitted two, Hearing and Seeing, more particularly for her service" (George Dalgarno, *Ars Signorum*, 1661).

38 From these coordinates, as well as the diagrams in **Figures 8** and **11**, it is notable that the vector ***Os*** points to the *geometric* mean of the system [√(**A•G**)], '*s*' (the 'mean green') being located diametrically opposite '**D**'. Similarly, the vector ***Op*** (the 'mean red') and the vector ***Ox*** (the 'mean violet') are located directly opposite the positions **A** and **G**, respectively the systems' *arithmetic* and *harmonic* means. One consequence of this is that not only is Newton's diagram symmetrical bilaterally (across the **Y** axis), the sonorific and colorific distributions are inversely symmetrical (very nearly) across the **X** axis. Moreover, Newton's solmization (*sol, la, fa, sol, la, mi, fa, sol*) assigned *sol* to the (least and most) extremes of the *spectrum* as well as the arithmetic mean (see also **Figure 7**). This usage was not impossible for the times, but it is certainly redundant and rather nonsensical musically. I suspect that Newton was enjoying primarily here a verbal '*sol-la*(r) centric' symbolism for the "Image of the Sun", the prismatic spectrum.

39 These 'forces' are simply assumed to be co-active in the sensorium; they are separated by neither time nor space, so no appeal to something as sophisticated as the 'inverse square law' is required in order to consider the nature of their interactions.

[40] "… the harmony and discord w^{ch} the more skillful Painters observe in colours may perhaps be effected & explicated by the various proportions of the æthereal vibrations as those of sounds are by the aereal" (Newton, "An Hypothesis Explaining the Properties of Light," 1675).

[41] Newton did not cultivate a strictly analogous 'wave theory of light' (see Shapiro 1973; 1993).

[42] "*Query* 12: Do not the Rays of Light in falling upon the bottom of the Eye excite vibrations in the Tunica Retina? Which Vibrations, being propagated along the solid Fibres of the optick Nerves into the Brain, cause the sense of seeing [?]."

"*Query* 13: Do not several sorts of Rays make Vibrations of several bignesses, which according to their bignesses excite Sensations of several Colours, much after the manner that the Vibrations of the Air, according to their several bignesses excite Sensations of several Sounds? And particularly do not the most refrangible Rays excite the shortest Vibrations for making a Sensation of deep violet, the least refrangible the largest for making a Sensation of deep red, and the several intermediate sorts of Rays, Vibrations of several intermediate bignesses to make Sensations of the several intermediate Colours?"

[43] "*Query* 14: May not the harmony and discord of Colours arise from the proportions [ratios] of the Vibrations propagated through the Fibres of the optick Nerves into the Brain, as the harmony and discord of Sounds arise from the proportions [ratios] of the Vibrations of the Air? For some Colours, if they be viewed together, are agreeable to one another, as those of Gold and Indigo, and other disagree." "If there were a theory of colour harmony, perhaps it would begin by dividing the colours into different groups and forbidding certain mixtures or combinations and allowing others; and, as in [tonal] harmony, its rules would be given no justification" (Wittgenstein 1977:III-91:28e). Many of Newton's findings and rules were highly justified and demonstrated, while others, such as this one, were much less so.

[44] Although Newton's musical/metrical geometry of perceptual color space strikes us now as very odd indeed (though hardly 'strangely mystical'), his technique sustained some principles that later 'moderns' are struggling to reconstitute. (One could point to, for example, ideas like 'simultaneous levels of scale' or the 'consilience of diverse disciplines' or the 'interplay of organic and mechanical consciousness'). Newton's measures attended *both* to questions of the continuum, which fostered appreciation of nature's real actualities, *and* the organization of those pertinent to human perceivers, that is, their virtual reality. In other words, *both* 'subjective' *and* 'objective' aspects of the phenomenon—the physicality of light and the psychology of its sensation, perception and pertinence to human beings —are included in the singular measuring 'rule.' The 'rule' addressed *both* the qualifiable *and* the quantifiable as they cohere within the employment of the rule itself. The technique of measurement accommodated *both* the variance within certain limits of solar radiation (the lights visible least to most refracted) *and* the quasi-discrete unequal magnitudes of the "eminent species" of that visibility, the gross colorific categories. Newton's predecessors, Copernicus and Kepler, had also grappled with these ideas:

> "Copernicus had discovered the *symmetry* of the universe, that is its *order*, which is grounded in the agreement between [the planets'] distance from the solar center and [their] period of revolution, but the precise reasons for [the] actual distances, the *arrangement* [of the planets] at such and such intervals, was not clear. To *symmetry*, the principle regulates the linear ordering, [Kepler] adds *eurhythmy of arrangement*, the principle regulating both the magnitude of the intervals and the closure of the linear order. And so [his] work demonstrates that 'nothing is superfluous' or without reason, neither the magnitude of the intervals nor the [finite] number of the planets. Everything in the universe can be governed by the plan that governs the whole. Copernican organicism overflows" (Hallyn 1990:186).

[45] Maxwell's model appealed to the analytical framework of centroids of equilateral triangles, succeeding at something Newton had dismissed: "I could never yet by mixing only two primary [homogeneal] Colours produce a perfect White. Whether it may be compounded of a mixture of *three taken at equal distances* in the circumference [of the color-mixing circle] I do not know, but of four or five I do not much question but it may. But these are Curiosities of little or no moment to the understanding the Phænomena of Nature" (Newton, *Opticks* [1704[1979:156-7, *emphasis added*).

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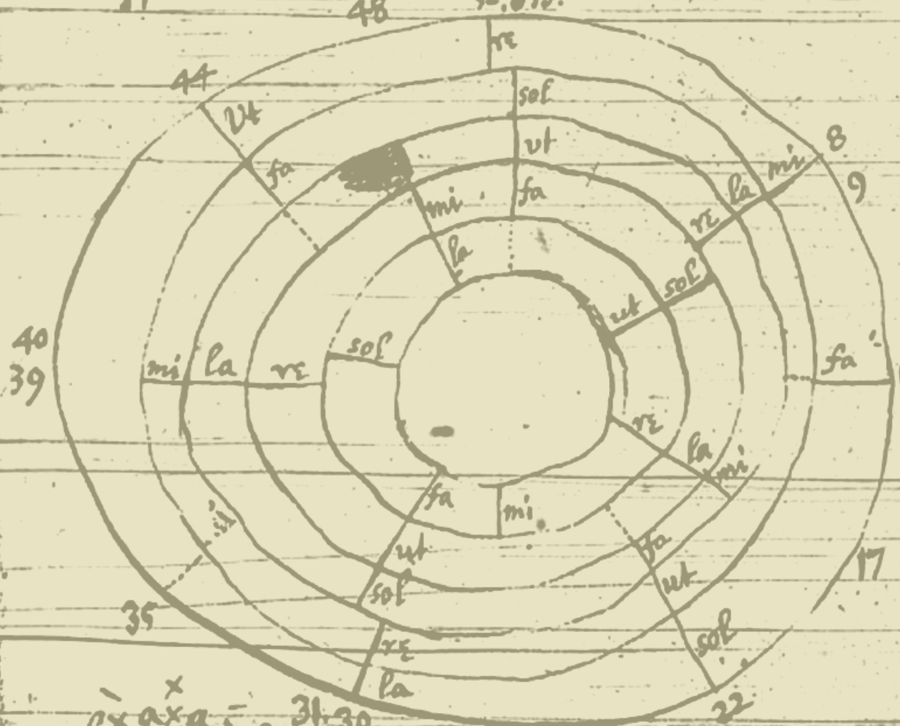
52:053.



for y^e ~~hyperbola~~ dm. suppose ak = ...

$ao = x$. $od = y$. $dc =$ asecant. $ca = v$. $og = z$.

$xy = aa$. $zy = ss - xx + 2vx - vv$ 110

$-a^4 + xxss - vvxx = x^4 + 2vx^3 = 0$ Double roots equall

$+2 \quad 0 \quad -2 \quad -1$ | $\dfrac{x^4 - a^4}{x^3} = v$.

$x - v = \dfrac{a^4}{x^3} = oc$. $od : oc :: hk : og$.

$\dfrac{aa}{x} . \dfrac{a^4}{x^3} :: a . z = \dfrac{aax}{x} = \dfrac{a^5}{x^3}$

$zxx = a^3$. wch equation continues y^e nature of y^e crooked line gh. Now supposing y^e line og always moves over y^e same superficies in y^e same time, it will increase in motion from kh in y^e same proportion y^t it decreaseth in lenght, y^e line re will move uniformly ~~through~~ from (mq) soe y^t y^e space $mqen = gokh$.

suppose $ok = a$. $ao = 2a$. $od = \dfrac{a}{2} = nm$. oe

$mqen = \frac{1}{2}aa = ogkh$.

Modi

1	2	1	2	1	1	2	1	2	1	1	2	1
3	4	3	4	3	2	3	3	4	3	2	3	3
4	5	6	6	5	4	5	4	5	5	4	5	4
6	7	8	7	7	6	7	6	7	6	6	7	6
8	9	9	9	8	8	9	8	9	8	7	8	8
9	10	10	11	10	9	10	10	11	10	9	10	9
11	11	11	12	12	11	12	11	12	12	11	12	11
	12											

1.6