JOS Plus

The Boonshaft and Fuchs Direct Reading Frequency Response Slide Rule; Part II

Richard Smith Hughes

The first nine pages are the instructions that came with my slide rule; I don't think they are complete, but it's all I have. The second part is the Appendix from my article and is an excellent overview of the slide rule.

BOONSHAFT AND FUCHS INC.

HATBORO INDUSTRIAL PARK • HATBORO • PENNSYLVANIA
TELEPHONE 215 OS 2-1240 TWX 215 675-1668

DIRECT READING FREQUENCY RESPONSE SLIDERULE

10 JUNE 1959

Abraham M. Fuchs

BOONSHAFT AND FUCHS INC.

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PART I - BASIC SET OF FUNCTION SCALES

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DIRECT READING FREQUENCY RESPONSE SLIDERULE

1- Introduction

The Direct Reading Frequency Response Sliderule provides a means for the rapid and accurate calculation of amplitude ratio and phase vs. frequency of a multiterm transfer function.

The Instruction Manual provides a detailed explanation of the operation of the sliderule.

The Direct Reading Frequency Response Sliderule was invented by Mr. Jens R. Jensen of the Danish Technical University in Copenhagen. He and Mr. M. Drost Larsen formed the Servo Calculator Company and are currently manufacturing the sliderule. Boonshaft and Fuchs, Inc. have the exclusive distributorship for the sliderule in the U.S.A. and Canada.

2- Basic Sliderule Components

The basic sliderule components are the Eleven Channel Sliderule Frame; Reversible Slider; Six Decade Frequency Scale; Leather Carrying Case and Function Scales.

2.1- Frequency Scale

The frequency scale covers six decades from .001 to 100. In use, the frequency unit may be considered as rad/sec., cycles per second, cycles per minute or any other desirable unit. Whatever value is selected for the frequency unit, it must be used throughout.

Each decade is subdivided into twenty frequencies. They are uniformly spaced on a log frequency scale. Thus they would plot at uniform spacing on the abscissa whenever the amplitude ratio or phase is plotted vs. log frequency. The frequencies are:

2- Basic Sliderule Components (cont'd.)

2.1- Frequency Scale (cont'd.)

1122	3548
1259	3981
1413	4467
1585	5012
1778	5623
1995	6310
2239	7079
2512	7943
2818	8913
3162	

(Note that $(1.122)^{20}=10$)

The sliderule gives a reading of phase and amplitude ratio at each of the above frequencies. This is far more than one would normally calculate by any other method.

Note that the frequency scale is duplicated on both sides.

2.2- Function Scale

Each function scale provides amplitude ratio on one side and phase on the other side of the scale for the function printed on the end of the scale.

Examine the $(1 + ju)$ and the $\frac{1}{1 + ju}$ scales. On one side is the amplitude ratio designated $200 \log (1 + ju)$ and $200 \log \frac{1}{1 + ju}$ respectively. The amplitude ratio of $(1 + ju)$ is printed in black indicating positive values and the amplitude ratio of $\frac{1}{1 + ju}$ is printed in red indicating negative values (amplitude ratio less than 1.0 or negative db).

2- Basic Sliderule Components (cont'd.)

2.2- Function Scale (cont'd.)

The break frequency where u = 1, or amplitude ratio = 3.0 db (30 centibels) is indicated on each scale by a bracket.

On the reverse side of these scales the phase angle is presented. It is noted as $(1 + ju)$ and $\frac{1}{1 + ju}$ respectively. Again, red is used for negative values (phase lag) and black is used for positive values (phase lead). The break frequency is bracketed (phase = 45°). Phase is presented to the nearest degree.

Placing these function scales alongside the frequency scale, it is evident that the spacing of the frequency values and that of the phase and amplitude ratio values is the same. Thus, when the function scales are set in position (see Section 3.2) with respect to the frequency scale, there is a one-to-one lineup of frequency numbers and amplitude ratio and phase numbers.

The other scales are color coded in red and black for negative and positive values as above. They also have amplitude ratio and phase on respective sides of the scale. Their break frequencies are bracketed and have values as listed below.

Function	Break Frequency	Amplitude Ratio - db	Phase Degrees
		(at the break frequency)	
$\frac{1}{(ju)^2 + 2\zeta ju + 1}$			
ζ = .1	u = 1	+ 14.0	- 90
= .2	u = 1	+ 8.0	- 90
= .3	u = 1	+ 4.4	- 90
= .4	u = 1	+ 1.9	- 90
= .5	u = 1	0.0	- 90

2- Basic Sliderule Components (cont'd.)

2.2- Function Scale (cont'd.)

Function	Break Frequency	Amplitude Ratio - db	Phase Degrees
		(at the break frequency)	
$\zeta = .6$	$u = 1$	-1.6	-90
$= .7$	$u = 1$	-2.9	-90
$= .8$	$u = 1$	-4.1	-90
$= .9$	$u = 1$	-5.1	-90
e^{-ju}	$u = 1$	0	-57°
$1/ju$	$u = 1$	0	-90°
ju	$u = 1$	0	$+90^\circ$
$1/(ju)^2$	$u = 1$	0	-180°
$(ju)^2$	$u = 1$	0	$+180^\circ$

2.3- Eleven Channel Frame

The Frame provides a means for holding the scales in the desired relative position. It securely holds each scale. It has provision for eleven scales. Each scale may be moved without in any way affecting the position of any other scale. The scales are held firmly enough so that the frame may be held in any orientation without the scales changing position.

The frame is transparent when viewed from top or bottom permitting the reading of both sides of the scales.

Note that the frame permits the study of any number of function scales up to ten.

2- Basic Sliderule Components (cont'd.)

2.4- Slider

The Slider can be positioned on either side of the frame by compressing its retaining spring against one edge of the frame and slipping the other side over the edge of the frame.

The slider is meant to be used on one side of the frame at a time. This permits the frame to lie flat on the desk while the slider is moved along the frame. To read the numbers on the other side of the function scales, the slider is repositioned to the other side of the frame.

The slider has two lines which bracket one row of data (one number from the frequency scale and one number from each function scale). Thus, the slider clearly outlines one row of amplitude ratio or phase data at that frequency.

2.5- Case

The leather case provides pockets for the several parts of the sliderule when it is not in use. It also has a brush to clean out the holder of any dirt which might collect there.

3- Operation

3.1- Frequency Scale

In computing the frequency response of a function using the Direct Reading Frequency Response Sliderule, one first selects a frequency scale. This might be radians per sec, cycles per sec, cycles per minute, etc. This scale must be continuously used throughout the setup of the function scales.

Insert the Frequency Scale in the Holder. While six decades of frequency are covered by the Scale, only five decades are effectively covered by the Holder. Therefore, place the frequency range of interest inside the Holder.

3- Operation (cont'd.)

3.2- Function Scales

Select the function scales which match the terms of the transfer function. The transfer function will almost always appear as a product of terms found on the scales. If cubic terms appear, they must be factored. Other functions not in the basic set of scales are available either as part of two added sets of standard scales or they can be obtained from Boonshaft and Fuchs, Inc. on special order. A list of those which have been manufactured to date, is available on request.

Each scale is inserted into the frame. They are turned, so that for all function scales, phase appears on one side of the holder and amplitude ratio on the other.

Each scale is positioned once for the entire computation. Its break frequency value is set in line with the nearest value to the break frequency as read on the frequency scale. The break frequency is computed as follows:

Scale	Function	Break Frequency rad/sec
$\frac{1}{ju+1}$	$\frac{1}{\tau j\omega+1}$	$\omega = 1/\tau$
$(ju+1)$	$(\tau j\omega+1)$	$\omega = 1/\tau$
$\frac{1}{ju}$, ju	$\frac{1}{\tau j\omega}$; $\tau j\omega$	$\omega = 1/\tau$
$\frac{1}{(ju)^2}$, $(ju)^2$	$\frac{1}{(\tau j\omega)^2}$; $(\tau j\omega)^2$	$\omega = 1/\tau$
$\left[\frac{1}{(ju)^2+2\zeta ju+1}\right]$	$\frac{1}{\left(\frac{j\omega}{\omega_n}\right)^2+2\zeta\frac{j\omega}{\omega_n}+1}$	$\omega = \omega_n$

3- Operation (cont'd.)

3.2- Function Scales (cont'd.)

Scale	Function	Break Frequency rad/sec
e^{-ju}	$e^{-\tau j\omega}$	$\omega = \frac{1}{\tau}$

When using a frequency scale of cycles/second (f),

$$f = \omega/2\pi$$

or if cycles/minute (F)

$$F = \frac{60\,\omega}{2\pi}$$

The slider should be used to facilitate the accurate setting of the function scales. Note that the function scales may be located on either the phase or amplitude ratio sides. The other side of the function scale will then be correctly positioned.

3.3- Totalizing

After the scales have been positioned, the overall amplitude ratio (or phase) is computed at each frequency by:

a- Place the brackets of the slider over the frequency.
b- Add the contribution of each term (remembering red is negative and black is positive) as shown within the brackets.
c- The total is the amplitude ratio (or phase) of the overall function at that frequency.

The values at other frequencies are obtained by repeating the above with the slider moved to the other frequencies.

4- Accuracy

4.1- Amplitude Ratio

Amplitude ratio is expressed in tenths of a db or centibels. Thus, the amplitude ratio values are rounded off to the nearest 0.1 db. Therefore, the maximum error per reading is $\pm$ .05 db.

The maximum probable error in amplitude ratio for ten scales is $\sqrt{10} \times (.05) = .158$ db. The maximum possible error using ten scales is .5 db.

4.2- Phase

Phase is expressed in degrees. Therefore, the maximum error per scale is $\pm .05^\circ$.

Using ten scales, the maximum probable error in phase is $\sqrt{10} \times 0.5^\circ = .158$ degrees and the maximum possible error is 5°.

In most instances, this phase error level will not be reached because

a- Less than 10 terms will be computed.
b- The phase angle of $1/ju$, ju, $(1/ju)^2$ and $(ju)^2$ is exact.

4.3- Break Frequency Location

Using 20 frequencies per decade, there is an 11% spread between two adjacent frequencies. Thus, when setting the break frequency of the function scales, they may be set as much as $\pm$ 5-1/2% from the desired value.

Usually this is not a significant error because the break frequency is not known this accurately. Frequently too, the study would cover a range of break frequencies. Additional scales are being made available to reduce this setting approximation in half for $(1 + ju)$ and $\frac{1}{(1 + ju)}$.

5- Overall Operation

The overall operation may be reduced to the following steps:

a- Set up the transfer function so that the terms are reduced to the forms available on the scales.

b- Select one scale for each term in the transfer function.

c- Place the frequency scale in the frame. Center the frequency range of interest.

d- Place the function scales in the frame. Use one side of the frame for phase and one for amplitude ratio.

e- Compute the break frequency of each term. Locate each scale so that its break frequency is set opposite the correct frequency.

f- The transfer function is now completely set-up on the sliderule. The resulting phase or amplitude ratio can be obtained at any frequency by placing the slider so that its lines bracket the frequency. The contribution of each term is read from its scale, and the total obtained by summing the black printed numbers and subtracting the red printed numbers.

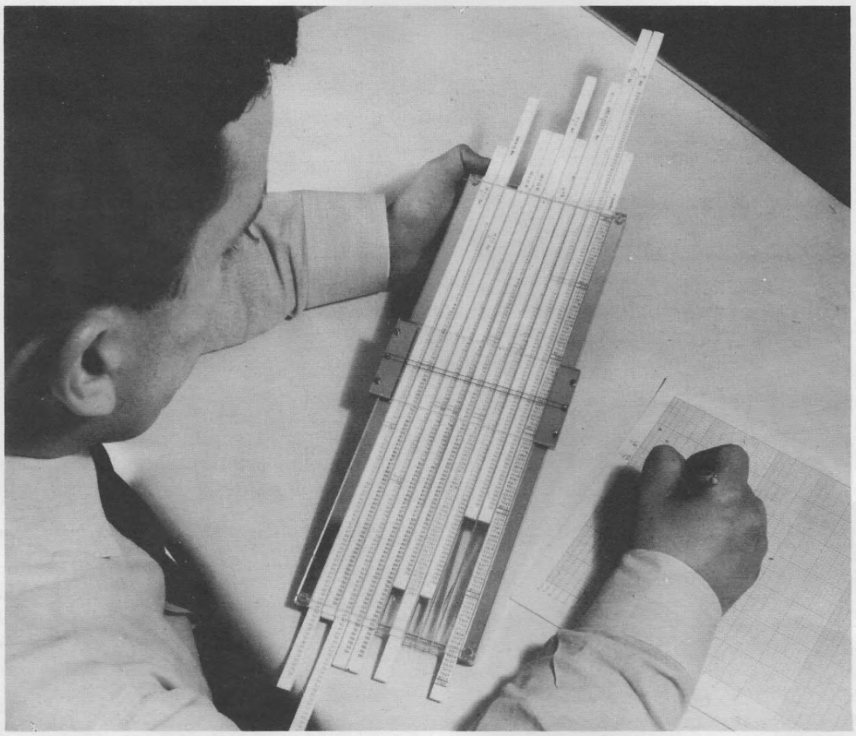
The results may be plotted term by term or overall.

Appendix
Basic Slide Rule Overview

BULLETIN 81407

BOONSHAFT AND FUCHS INC.

DIRECT READING FREQUENCY RESPONSE SLIDERULE*



Advantages offered by this NEW engineering tool . . .

1. Ease of Setup—A complex transfer function of up to ten terms is completely set up over the entire frequency range with only one setting of each scale.

2. Speed of Operation—Each scale is direct reading. No time is needed for interpolation or for determining scale factors. The values shown on the slider are discrete numbers.

3. High Accuracy—Values of phase angle are printed in degrees accurate to $\pm\frac{1}{2}°$, and of amplitude ratio in 0.1 db accurate ± 0.05 db. Scales are not crowded. They are easily read. Numbers are printed in red and black so that plus and minus values cannot be misread.

4. Flexibility—The sliderule can handle up to ten terms at one setting. Individual terms are easily changed or moved as to the location of their break frequency. The frequency scale covers a useful range of over 100,000: 1 with one setting.

5. Simplicity—The sliderule can readily be used by technicians with a minimum of training.

*Patent applied for.

DESCRIPTION

The Direct Reading Frequency Response Sliderule provides a means for the simple, rapid, and simultaneous determination of the amplitude ratio and phase vs. frequency of all of the terms of a complex transfer function. The amplitude ratio and phase information is presented as numbers for direct readout at 20 frequency intervals per decade through six decades. Because one set-up of the sliderule gives all of the data on the amplitude ratio and phase vs. frequency of a transfer function, the sliderule facilitates the study of the effects of variation in system parameters, the optimization of compensating networks, and the effects of high frequency (including quadratic) terms.

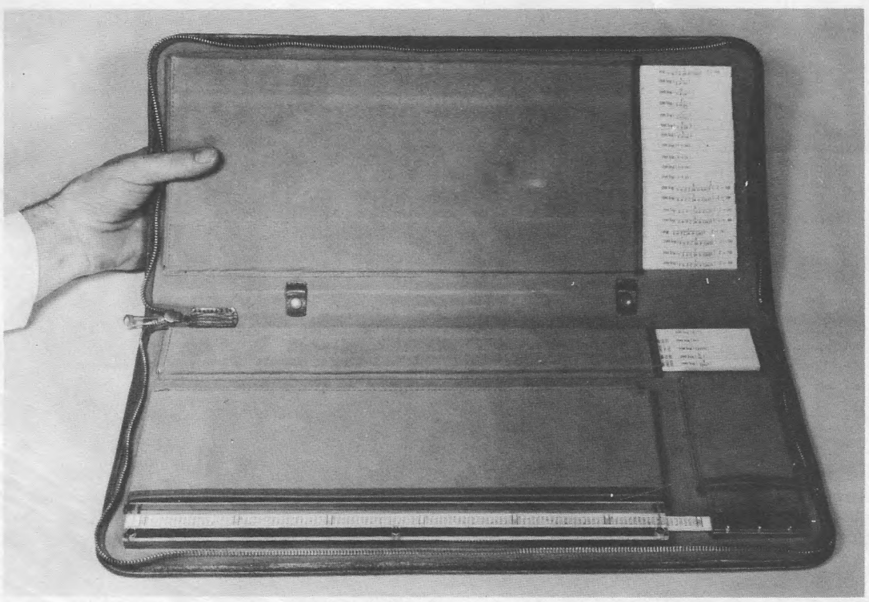
This frequency response computing aid is an eleven channel sliderule. One channel holds a frequency scale covering six decades from 0.001 to 1,000. Each of the ten remaining channels holds a scale which represents one term of the transfer function. Twenty-four such function scales are furnished.

The frequency scale covers six decades with each decade subdivided into twenty discrete frequencies uniformly spaced on a "log frequency" scale. These frequencies are printed on both sides of the scale.

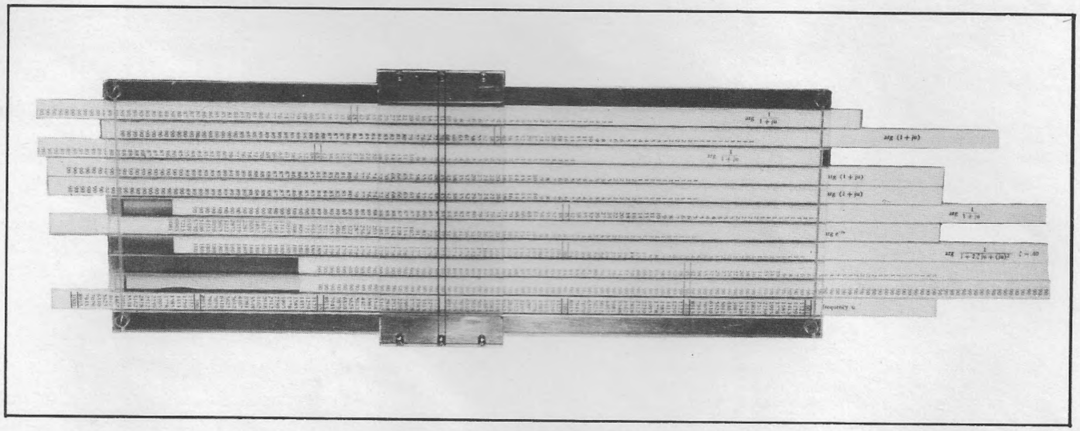
The function scales give amplitude ratio corresponding to these frequency values on one side and phase on the reverse side. Positive values of amplitude ratio and phase are shown in black and negative values in red.

To facilitate locating the function scales with respect to the frequency scale, the phase and amplitude ratio values at the "break" frequency ($u = 1.0$) are clearly bracketed by a pair of lines. Phase values are shown to the nearest degree. Amplitude ratio is expressed in 0.1 db units.

The eleven channel sliderule frame holds each scale separately. Thus each scale may be independently adjusted without disturbing the other scales. The clear plastic top and bottom covers permit reading both amplitude ratio and phase with one setting. The slider has two cursors running the full width of the frame and spaced to bracket only one number on each scale. The slider can be moved from one side of the sliderule reading amplitude ratio to the other reading phase. The contribution of each term in the transfer function to the overall phase or amplitude ratio at each frequency may be read individually and summed with one setting of the slider.



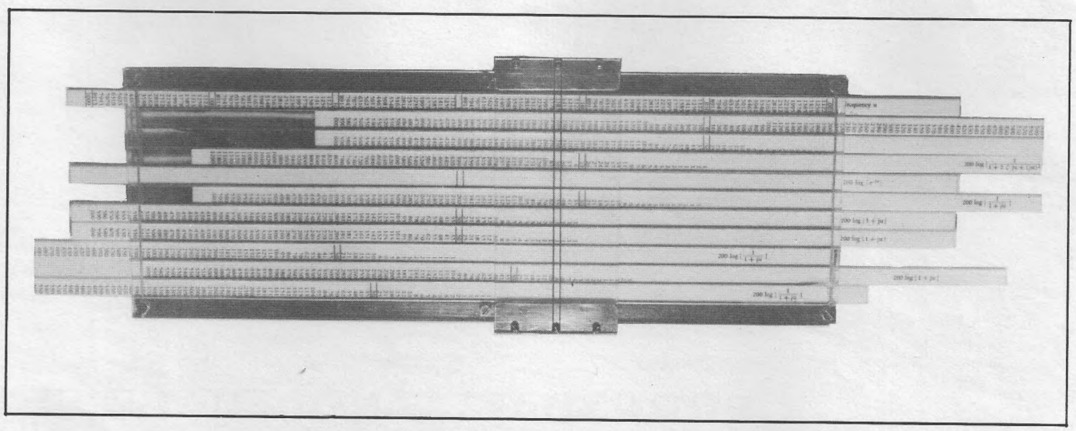
The complete Direct Reading Frequency Response Sliderule shown in convenient leather carrying case.



Slider positioned for reading phase angle.

OPERATION

1. Place the frequency scale in the frame so that the desired frequency range is centered. The frequency scale may be read as rad/sec., cycles/sec., or cycles/min., etc.
2. Choose the scales which have the terms of the transfer function. Place each scale in the frame using one side of the frame for phase and one for amplitude ratio.
3. Locate each scale so that the break frequency is set opposite the correct frequency.
4. The transfer function is now completely set up on the sliderule. The resulting phase and amplitude ratio can be obtained at any frequency by placing the slider so that its lines bracket that frequency. The contribution of each term is read from its scale, and the total obtained by summing the black printed numbers and subtracting the red printed numbers. The results may be plotted term by term or overall.



Reverse side of above illustration showing the slider positioned for reading amplitude ratio.

APPLICATIONS

1. Stability studies of Feedback Control Systems
2. Effect of Variation in system parameters on performance
3. Design and Optimization of Compensating Networks and Controller functions
4. Evaluation of second order effects of high frequency terms such as time delays and low damped quadratics (such as occur in hydraulic controls)
5. Study of Non-Linear Systems using the Describing Function technique

The Direct Reading Frequency Response Sliderule was developed by J. R. Jensen and M. Drost Larsen of the Servo Calculator Co. of Denmark.

SPECIFICATIONS

The Direct Reading Frequency Response Sliderule as furnished includes:

* Eleven channel sliderule frame
* Reversible slider
* Six decade frequency scale
* Leather carrying case for all parts
* Instruction manual
* Twenty-four individual function scales as follows:

$(1 + ju)$ — 4 scales

$\frac{1}{(1 + ju)}$ — 6 scales

ju — 1 scale

$(ju)^2$ — 1 scale

$\frac{1}{ju}$ — 1 scale

$\frac{1}{(ju)^2}$ — 1 scale

e^{-ju} — 1 scale

and $\frac{1}{1 + 2\zeta ju + (ju)^2}$

One scale each where

$\zeta = 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9.$

PRICE: $170.00 f.o.b.
Hatboro, Pennsylvania

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