JOS Plus

The Faber-Castell 2/84N Mathema; Scale Operation

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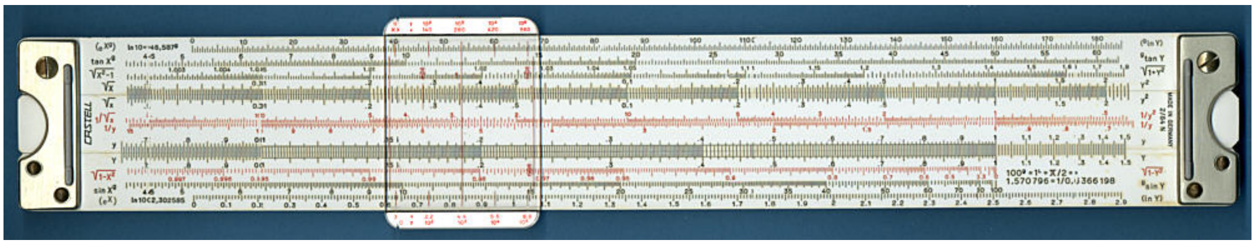
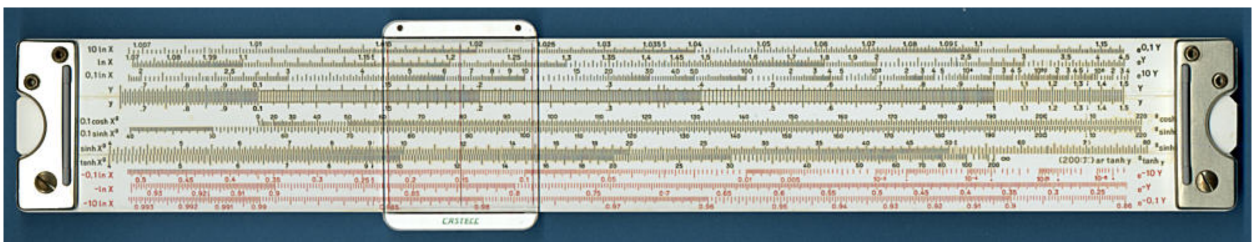
This short article is dedicated to the memory of Bill Robinson, my friend and ***the*** *expert on all Hyperbolic slide rules.*





http://www.tcocd.de/Pictures/SlideRules/FaberCastell/2-84n.shtm

Trigonometric Scales

sin X^g (^{g}sin Y)

This scale is our typical sin/sin^{-1} scale, however without the cos scale! The √(1-x^2) scale is used instead;. Note 90^o = 100^g = π/2 Radian.

X^g on the sin X^g scale = sin X^g on the Y scale = cos X^g on the √(1-x^2) scale

sin X^g on the Y scale = X^g on the sin X^g scale

cos X^g on the √(1-x^2) scale = X^g on the sin X^g scale

On the lower right side we have;

100^g = 1^L *(L is probably the symbol for 100^g)* = π/2 *(π/2 Radians/100^g)*

= 1.570796 *(Radians/100^g = 0.01570796 Rad./g)* = 0.6366197 *(100^g/Radian = 63.66197 g/Rad.)*

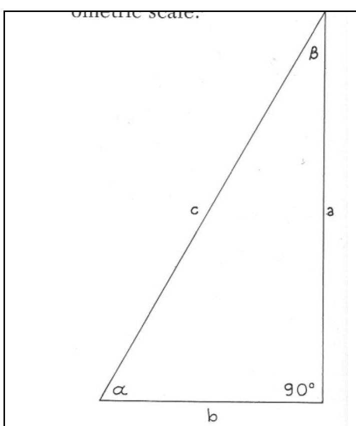
tan X^g (^{g}tan Y)

Typical tan/$\tan^{-1}$ scale, however only for $4.2^g \rightarrow 61^g$. For $X^g > 50^g$, use the 1/y scale (with the slide closed, y/Y = 1) and $100^g - X^g$ on the tan scale. An inverted tan scale for $X^g > 50^g$ would have been useful.

Pythagorean (P) Scales

$\sqrt{(1 - x^2)}$ $(\sqrt{(1 - Y^2)})$

This scale is discussed above, and was used on several Faber-Castell slide rules, labeled as the P scale. It can also be used is solving for one side, and the angles, of a right triangle knowing the hypotenuse and other side; I find it much easier using the trig scales on my K&E.

 I couldn't find a figure with 100^g	Using the Trig scales **c and a known** $\alpha = \sin^{-1}(a/c)$; $b = (c)\cos\alpha$ **c and b known** $\alpha = \cos^{-1}(b/c)$; $a = b/\sin\alpha$ **a and b known** $\alpha = \tan^{-1}(b/a)$; $c = b/\sin\alpha = a/\cos\alpha$ $\beta^g = 100 - \alpha^g$

Using the $\sqrt{(1 - x^2)}$ $(\sqrt{(1 - Y^2)})$, P, Scale

Knowing c and a or b; solves for b or a and the angles

$c^2 = a^2 + b^2$ $a = c\sqrt{((1 - (b^2/c^2)))}$ $b = c\sqrt{((1 - (a^2/c^2)))}$

Procedure	c = 5, a = 4	c = 5, b = 3
1)Place c on y scale over right Y index; c is greator than a or b	c = 5 over Y index	c = 5 over Y index
2)Place cursor over a or b on y scale	Cursor over a = 4 on y scale	Cursor over b = 3 on y scale
3)Read angle X_1^g on the sin scale (we don't know if it's α or β yet)	Read $X_1^g = 59^g$ on sin scale	Read $X_1^g = 41^g$ on sin scale
4)Under cursor read $\sqrt{(1 - x^2)}$ value	$\sqrt{(1 - x^2)} = 0.6$	$\sqrt{(1 - x^2)} = 0.8$
5)Place $\sqrt{(1 - x^2)}$ value on the Y scale	0.6 on the Y scale	0.8 on the Y scale
6)Read b or a on the y scale	Read b = 3 on y scale under cursor	Read a = 4 on y scale under cursor
7)Read and angle X_2^g on the sin scale	$X_2^g = 41^g$ on sin scale	$X_2^g = 59^g$ on sin scale
If a > b, $\alpha > 50^g$ If b > a, $\alpha < 50^g$	a > b; $X_1^g = \alpha = 59^g$ $X_2^g = \beta = 41^g$	a > b; $X_2^g = \alpha = 59^g$ $X_1^g = \beta = 41^g$

Using the $\sqrt{(X^2 - 1)}$ ($\sqrt{(1 + Y^2)}$) Scale

This scale solves for the hypotenuse, c, and angles knowing a and b; it is much easier using the Trig scales.

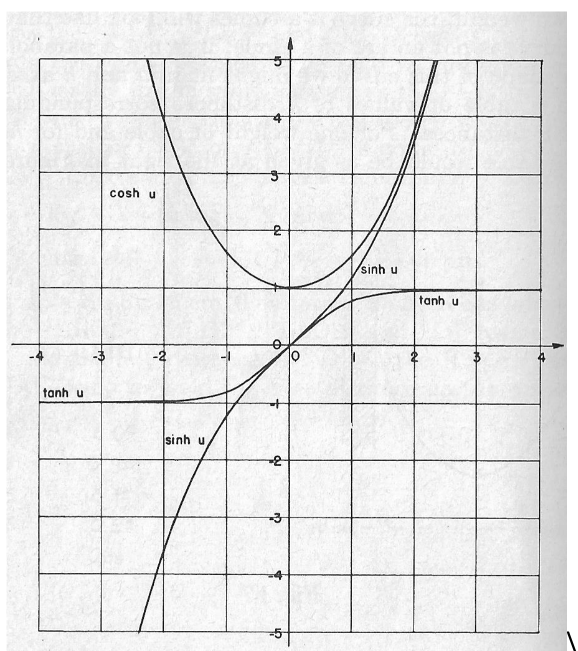
$$c^2 = a^2 + b^2 \quad c = b\sqrt{((1 + (a^2/b^2)))} = a\sqrt{((1 + (b^2/a^2))}$$

Procedure	a = 4, b = 3	a = 3, b = 4
1)Place larger of a or b on y scale over Y = 1	a = 4 over Y = 1	b = 4 over Y = 1
2)move cursor over the smaller b or a on y scale	Cursor to 3 on y scale	Cursor to 3 on y scale
3)Read X^g on the tan scale under the cursor; If a > b, $\alpha > 50^g$, b > a, $\beta > 50^g$	$X^g = 41^g$ a > b, thus $X^g = 41^g = \beta$ $\alpha = 100 - 41^g - 59^g$	$X^g = 41^g$ b > a, thus $X^g = 41^g = \alpha$ $\beta = 100 - 41^g = 59^g$
4)Read $\sqrt{(X^2 - 1)}$ value and	$\sqrt{(X^2 - 1)} = 1.25$	$\sqrt{(X^2 - 1)} = 1.25$
5)Place cursor over $\sqrt{(X^2 - 1)}$ value on the right extended Y scale	1.25 on Y scale	1.25 on Y scale
6) Read c on the y scale under the cursor	Read c = 5 on the y scale	Read c = 5 on the y scale

Hyperbolic Functions

The figure below may be of help in the following discussion.

Hyperbolic Functions



←←← u →→→

The x axis is in Radians, u; 1 Radian = 63.66 Grad

0.1 cosh X^g (^{g}cosh 10y))

With X^g on the 0.1 cosh X^g scale we must multiply the y scale reading by 10

$X^g > 200^g$, cosh $X^g \approx e^{X^g}/2$

$X^g < 10^g$, cosh $X^g \approx 1$

0.1 sinh X^g (^{g}sinh 10y))

With X^g on the 0.1 sinh X^g scale we must multiply the y scale reading by 10

$X^g > 220^g$, sinh $X^g \approx e^{X^g}/2$

sinh X^g (^{g}sinh y))

$X^g < 5^g$, sinh $X^g \approx 0.0157\ X^g$

Tanh X^g (^{g}tanh y); Note it should be ^{g}tanh Y

$X^g > 150^g$, tanh $X^g \approx 1$

$X^g < 5^g$, tanh $X^g \approx 0.0157X^g$

On the right side we have;

200 : π (*200/π = 200^g/π Radians, or 63.66g/Radian*) = ^{g}tanh y *(should be ^{g}tanh Y)*

e^X (LnY)

Using the e^X (Ln Y) scales gives better accuracy than the LL scales

e^{+X}

$0 \leq +X \leq 11.7$ $\qquad$ $1 \leq (e^X) \leq 120.6(10^3)$

e^{-X}

$-11.7 \leq -X \leq 0$ $\qquad$ $8.3(10^{-6}) \leq e^{-X} \leq 1$

e^X Cursor (use x for the cursor)

Let $e^{(+/-)X} = e^{|X|}$

|X| limits	Cursor x hairline	Cursor x value	e^X scale multiplier	e^{-X} scale multiplier
0 → 2.2	X 10	0	$10^{(1)}$	$10^{(-1)}$
2.2 → 4.4	2.2 10^2	2.2	10^2	10^{-2}
4.4 → 6.6	4.4 10^3	4.4	10^3	10^{-3}
6.6 → 8.8	6.6 10^4	6.6	10^4	10^{-4}
8.8 → (11.7)	8.8 10^5	8.8	10^5	10^{-5}

The basic procedures for solving e^X and e^{-X} are the same; use |X| for +/-X

Let +/- X = |X|	$e^{+/-(0.5)}$	$e^{+/-(7)}$	$e^{+/-(11)}$
1)Determine the |X| limits from the Table	|X| limits 0 → 2.2	|X| limits 6.6 → 8.8	|X| limits 8.8 → (11.7)
2)Knowing the |X| limits determine which cursor hairline to use from Table	Use the x/10 cursor	Use the $6.6/10^4$ cursor	Use the $8.8/10^5$ cursor
3) Calculate |X| - x (the cursor value)	|X| - x = 0.5 – 0 = 0.5	|X| - x = 7 – 6.6 = 0.4	|X| - x = 11 – 8.8 = 2.2
4) Place the x cursor hairline over |X| - x on the e^X scale	Cursor x/10 hairline over 0.5	Cursor $6.6/10^4$ hairline over 0.4	Cursor $8.8/10^5$ hairline over 2.2
	e^X		
e^X = Y scale reading multiplied by the cursor multiplier; $e^X = Y(10^x)$	Y = 0.165 $e^{0.5} = 0.125(10) = 1.65$	Y =0.1095 $e^7 = 0.1095(10^4) = 1.095(10^3)$	Y = 0.599 $e^{11} = 0.599(10^5)$ $= 59.9(10^3)$
	e^{-X}		
Close the slide; y/Y = 1 **e^{-X}** = 1/y scale reading times the cursor multiplier; $e^{--X} = (1/y)(10^{-x})$	1/y = 6.05 $e^{-0.5} = 6.05(10^{-1}) = 0.605$	1/y = 9.11 $e^{--7} = 9.11(10^{--4}) = 911\ (10^{-6})$	1/y = 1.67 $e^{-11} = 1.67(10^{-5})$ $= 16.7(10^{-6})$

Ln Y

$1 \le Y \le 120.6(10^3)$ $\quad$ $0 \le LnY \le 11.7$

The left side of the e^X (LnY) scale has Ln 10 = 2.302585

1)Write Y as $0.YYY(10^x)$	$Y = 10 = 0.1(10^2)$	$Y = 1.095(10^3) = 0.1095(10^4)$
2)The exponent 10^x tells which cursor to use	10^2; use $2.2/10^2$ cursor	10^4; use $6.6/10^4$cursor
3)Place primary hairline over 0.YYY on the Y scale	Primary hairline over 0.1 on the Y scale	Primary hairline over 0.1095 on the Y scale
4)Read e^X under the proper	Under the X/10 hairline	Under the $6.6/10^4$ hairline

cursor hairline	read 0.102 on the e^X scale	read 0.4 on the e^X scale
5)Add e^X value to cursor value x	0.102 + 2.2 = 2.302	0.4 + 6.6 = 7
6) Ln Y = e^X value + the cursor value x	Ln 10 = 2.302	Ln $1.095(10^3) = 7$

$8.36(10^{-6}) \leq Y \leq 1 \qquad -11.7 \leq \text{Ln}Y \leq 0$

1)Write Y as $Y.YY(10^{-x})$	$Y = 0.00008 = 8(10^{-5})$	$Y = 0.5 = 5(10^{-1})$
2)The exponent 10^{-x} tells which cursor to use	(10^{-5}); use $8.8/10^5$ hairline	(10^{-1}); use x/10 hairline
Close the slide; y/Y = 1 3)Place primary hairline over Y.YY on the 1/y scale	Primary hairline over 8 on the 1/y scale	Primary hairline over 5 on the 1/y scale
4)Read e^X under the proper cursor hairline	Under the $8.8/10^5$ hairline read 0.63 on the e^X scale	Under the x/10 hairline read 0.691 on the e^X scale
5)Add e^X value to cursor value x	0.63 + 8.8 = 9.43	0.691 + 0= 0.691
6) Ln Y = - [e^X value + the cursor value x]	Ln (0.00008) = - 9.43	Ln (0.5) = - 0.691

$e^{(+/-)Xg}$ (^{g}LnY)

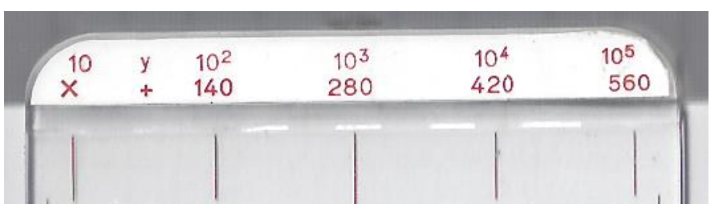
Note; We don't need this scale/cursor! Just convert X^g to radians (multiply X^g by π/200 = 0.0157Rad/grad) and use the e^x scale and cursor. Here is the e^{xg} operation for those interested.

e^{+Xg}

$0 \leq +X^g \leq 745^g \qquad 1 \leq (e^{Xg}) \leq 120.6(10^3)$

e^{-Xg}

$-745^g \leq -X^g \leq 0 \qquad 8.3(10^{-6}) \leq e^{-Xg}) \leq 1$



e^{Xg} Cursor (use x for the cursor)

Oet $e^{(=/-)Xg} = e^{|Xg|}$

$\lvert X^g \rvert$ limits	Cursor x hairline	Cursor x value	e^{Xg} scale multiplier	e^{-Xg} scale multiplier
$0 \to 140^g$	$10^{(1)}$ x	0	$10^{(1)}$	$10^{(-1)}$
$140^g \to 280^g$	10^2 140	140^g	10^2	10^{-2}
$280^g \to 420^g$	10^3 280	280^g	10^3	10^{-3}
$420^g \to 560^g$	10^4 420	420^g	10^4	10^{-4}
$560^g \to (745^g)$	10^5 560^g	560^g	10^5	10^{-5}

The basic procedures for solving e^{Xg} and e^{-Xg} are the same; use $|X^g|$ for $+/-X^g$

Let (+/-) X_g = $\|X^g\|$	$e^{+/-(110g)}$	$e^{+/-(300g)}$	$e^{+/-(720g)}$
1)Determine the $\|X^g\|$ limits from the Table	$\|X^g\|$ limits $0 \rightarrow 140^g$	$\|X^g\|$ limits $280^g \rightarrow 420^g$	$\|X^g\|$ limits $560 \rightarrow (745)$
2)Knowing the $\|X^g\|$ limits determine which cursor hairline to use from Table	Use the 0/x cursor	Use the $10^3/280$ cursor	Use the $10^5/560$ cursor
3) Calculate $\|X^g\|$ - x (the cursor value)	$\|X^g\| - x = 110 - 0 = 110$	$\|X^g\| - x = 300 - 280 = 20$	$\|X^g\| - x = 720 - 560 = 160$
4) Place the x cursor hairline over $\|X^g\|$ - x on the e^{Xg} scale	Cursor 10/x hairline over 110^g	Cursor $10^3/280$ hairline over 20	Cursor $10^5/560$ hairline over 160
e^{Xg}			
e^{Xg} = Y scale reading multiplied by the cursor multiplier; $e^{Xg} = Y(10^x)$	Y = 0.564 $e^{+110g} = 0.564(10) = 5.64$	Y =0.111 $e^{+300g} = 0.111(10^3) = 111$	Y = 0.815 $e^{+720g} = 0.815(10^5) = 81.5(10^3)$
e^{-Xg}			
Close the slide; y/Y = 1 **e^{-Xg}** = 1/y scale reading times the cursor multiplier; $e^{--Xg} = (1/y)(10^{-x})$	1/y = 1.775 $e^{-110g} = 1.775(10^{-1}) = 0.1775$	1/y = 9 $e^{-300g} = 9(10^{--3})$	1/y = 1.225 $e^{-720g} = 1.225(10^{-5}) = 12.25(10^{-6})$

^{g}Ln Y

$1 \leq Y \leq 120.6(10^3)$ $\quad\quad$ $0 \leq {}^g\text{Ln } Y \leq 745^g$

The left side of the e^{Xg} (^{g}LnY) scale has Ln 10 = 146.587^g; it should be ^{g}Ln 10

1)Write Y as $0.YYY(10^x)$	$Y = 10 = 0.1(10^2)$	$Y = 81.5(10^3) = 0.815(10^5)$
2)The exponent 10^x tells which cursor to use	10^2; use $10^2/140$ cursor	10^5; use $10^5/560$ cursor
3)Place primary hairline over 0.YYY on the Y scale	Primary hairline over 0.1 on the Y scale	Primary hairline over 0.815 on the Y scale
4)Read e^{Xg} under the proper Cursor hairline	Under the $10^2/140$ hairline read 6.66 on the e^{Xg} scale	Under the $10^5/560$ hairline read 160 on the e^{Xg} scale
5)Add e^{Xg} value to cursor value x	6.66 + 140 = 146.6	560 + 160 = 720
6) ^{g}Ln Y = e^{Xg} value + the cursor value x	^{g}Ln 10 = 146.6	^{g}Ln $81.5(10^3)$ = 720

$8.36(10^{-6}) \le Y \le 1 \quad -11.7 \le LnY \le 0$

1)Write Y as Y.YY(10^{-x})	Y = 12.3(10^{-6}) = 1.23(10^{-5})	Y = 0.178 = 1.78(10^{-1})
2)The exponent 10^{-x} tells Which cursor to use	(10^{-5}); use 10^5/560 hairline	(10^{-1}); use 10/x hairline
Close the slide; y/Y = 1 3)Place primary hairline over Y.YY on the 1/y scale	Primary hairline over 1.23 on the 1/y scale	Primary hairline over 1.78 on the 1/y scale
4)Read e^{Xg} under the proper Cursor hairline	Under the 10^5/560 hairline read 159.7 on the e^{Xg} scale	Under the 10/x hairline read 110 on the e^{Xg} scale
5)Add e^{Xg} value to cursor value x	159.7 + 560 = 719.7	110 + 0 = 110
6) ^{g}Ln Y = - [e^{Xg} value + the cursor value x]	^{g}Ln (0.00008) = - 719.7	^{g}Ln (0.178) = - 110

The Cursor

Grad/Radian, Radian/Grad conversion

$\pi/2$

(2/π)Not Shown

1 (g) Primary Hairline R

Grad Radian

Grads to Radian

$$X^R = (X^g)(\pi/200) = (X^g)(\pi/2)(1/100)$$

Procedure	$X^g = 1$	$X^g = 30$	$X^g = 100$
1)g hairline over X^g on the Y scale	g hairline over $X^g = 1$	g hairline over $X^g = 30$	g hairline over $X^g = 100$
2)X^R = (R reading on Y scale)/100	$X^R = 1.15/100 = 0.0157$	$X^R = 47.1/100 = 0.471$	$X^R = 157/100 = 1.57$

Radians to Grads

$$X^g = (X^R)(200/\pi) = (X^R)(2/\pi)(100)$$

Procedure	$X^R = 1$	$X^R = 0.5$	$X^R = 0.03$
1)R hairline over X^R on the Y scale	R hairline over $X^R = 1$	g hairline over $X^g = 0.5$	g hairline over $X^g = 0.03$
2)X^g = (g reading on Y scale)(100)	$X^g = 0.636(100) = 63.6$	$X^g = 0.319(100) = 31.9$	$X^g = 0.191(10) = 1.91$

π/4

The area of a circle = $(\pi d^2/4) = (\pi/4)d^2$

1) d^2 on the Y scale

2) Read area under $\pi/4$ on the $\sqrt{X}$ scale